Chapter 8

Multielement Arrays

The gain and directivity offered by an array of elements represents a worthwhile improvement both in transmitting and receiving. Power gain in an antenna is the same as an equivalent increase in the transmitter power. But unlike increasing the power of one's own transmitter, antenna gain works equally well on signals received from the favored direction. In addition, the directivity reduces the strength of signals coming from the directions not favored, and so helps discriminate against interference.

One common method of obtaining gain and directivity is to combine the radiation from a group of $\lambda/2$ dipoles to concentrate it in a desired direction. A few words of explanation may help make it clear how power gain is obtained.

In **Fig 1**, imagine that the four circles, A, B, C and D, represent four dipoles so far separated from each other that the coupling between them is negligible. Also imagine that point P is so far away from the dipoles that the distance from P to each one is exactly the same (obviously P would have to be much farther away than it is shown in this drawing). Under these conditions the fields from all the dipoles will add up at P if all four are fed RF currents in the same phase.

Let us say that a certain current, I, in dipole A will produce a certain value of field strength, E, at the distant point P. The same current in any of the other dipoles will produce the same field at P. Thus, if only dipoles A and B are operating, each with a current I, the field at P will be 2E. With A, B and C operating, the field will be 3E, and with all four operating with the same I, the field will be 4E. Since the power received at P is proportional to the square of the field strength, the relative power received at P is 1, 4, 9 or 16, depending on whether one, two, three or four dipoles are operating.

Now, since all four dipoles are alike and there is no coupling between them, the same power must be put into each in order to cause the current I to flow. For two dipoles the relative power input is 2, for three dipoles it is 3, for four dipoles 4, and so on. The actual gain in each case is the relative received (or output) power divided by the relative input power. Thus we have the results shown in **Table 1**. The power ratio is directly proportional to the number of elements used.

It is well to have clearly in mind the conditions under which this relationship is true:

1) The fields from the separate antenna elements must be in phase at the receiving point.
2) The elements are identical, with equal currents in all elements.
3) The elements must be separated in such a way that the current induced in one by another is negligible; that is, the radiation resistance of each element must be the same as it would be if the other elements were not there.

Very few antenna arrays meet all these conditions exactly. However, the power gain of a directive array using dipole elements with optimum values of element spacing is approximately proportional to the number of elements.

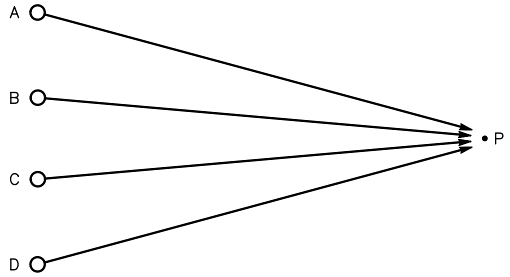


Fig 1—Fields from separate antennas combine at a distant point, P, to produce a field strength that exceeds the field produced by the same power in a single antenna.

Table 1
Comparison of Dipoles with Negligible Coupling (See Fig 1)

Dipoles	*Relative Output Power*	*Relative Input Power*	*Power Gain*	*Gain in dB*
A only	1	1	1	0
A and B	4	2	2	3
A, B and C	9	3	3	4.8
A, B, C and D	16	4	4	6

Another way to say this is that a gain of approximately 3 dB will be obtained each time the number of elements is doubled, assuming the proper element spacing is maintained. It is possible, though, for an estimate based on this rule to be in error by a ratio factor of two or more (gain error of 3 dB or more), especially if mutual coupling is *not* negligible.

DEFINITIONS

An *element* in a multi-element directive array is usually a λ/2 radiator or a λ/4 vertical element above ground. The length is not always an exact electrical half or quarter wavelength, because in some types of arrays it is desirable that the element show either inductive or capacitive reactance. However, the departure in length from resonance is ordinarily small (not more than 5% in the usual case) and so has no appreciable effect on the radiating properties of the element.

Antenna elements in multi-element arrays of the type considered in this chapter are always either *parallel*, as in **Fig 2A**, or *collinear* (end-to-end), as in Fig 2B. Fig 2C shows an array combining both parallel and collinear elements. The elements can be either horizontal or vertical, depending on whether horizontal or vertical polarization is desired. Except for space communications, there is seldom any reason for mixing polarization, so arrays are customarily constructed with all elements similarly polarized.

A *driven element* is one supplied power from the transmitter, usually through a transmission line. A *parasitic element* is one that obtains power solely through coupling to another element in the array because of its proximity to such an element.

A *driven array* is one in which all the elements are driven elements. A *parasitic array* is one in which one or more of the elements are parasitic elements. At least one element must be a driven element, since you must somehow introduce power into the array.

A *broadside array* is one in which the principal direction of radiation is perpendicular to the axis of the array and to the plane containing the elements, as shown in **Fig 3**. The elements of a broadside array may be collinear, as in Fig 3A, or parallel (two views in Fig 3B).

An *end-fire array* is one in which the principal direction of radiation coincides with the direction of the array axis. This definition is illustrated in **Fig 4**. An end-fire array must consist of parallel elements. They cannot be collinear, as λ/2 elements do not radiate straight off their ends. A Yagi is a familiar form of an end-fire array.

A *bidirectional array* is one that radiates equally well in either direction along the line of maximum radiation. A bidirectional pattern is shown in **Fig 5A**. A *unidirectional array* is one that has only one principal direction of radiation, as the pattern in Fig 5B shows.

The *major lobes* of the directive pattern are those in which the radiation is maximum. Lobes of lesser radiation intensity are called *minor lobes*. The *beamwidth* of a directive antenna is the width, in degrees, of the major lobe between the two directions at which the relative radiated power is equal to one half its value at the peak of the lobe.

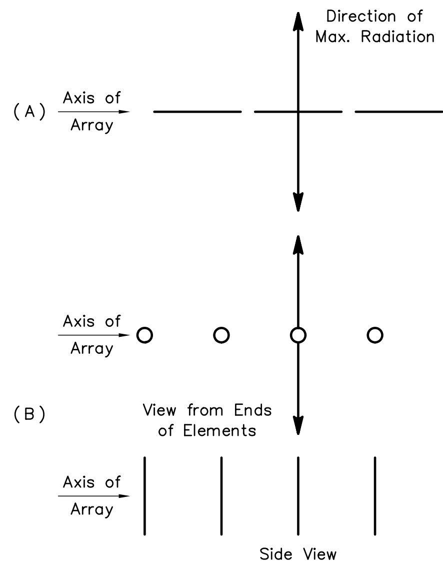


Fig 3—Representative broadside arrays. At A, collinear elements, with parallel elements at B.

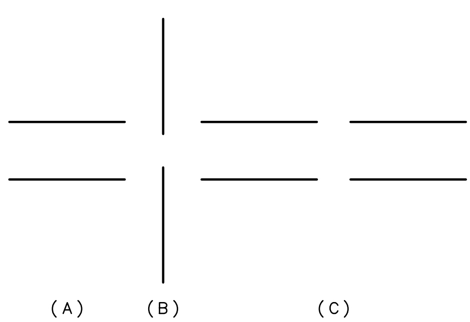


Fig 2—At A, parallel and at B, collinear antenna elements. The array shown at C combines both parallel and collinear elements.

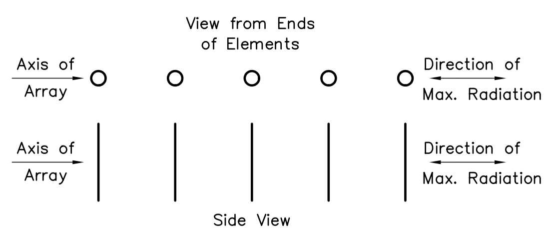


Fig 4—An end-fire array. Practical arrays may combine both broadside directivity (Fig 3) and end-fire directivity, including both parallel and collinear elements.

At these *half-power points* the field intensity is equal to 0.707 times its maximum value, or down 3 dB from the maximum. **Fig 6** shows a lobe having a beamwidth of 30°.

Unless specified otherwise, the term *gain* as used in this section is the power gain over an isotropic radiator in free space. The gain can also be compared with a λ/2 dipole of the same orientation and height as the array under discussion, and having the same power input. Gain may either be measured experimentally or determined by calculation. Experimental measurement is difficult and often subject to considerable error, for two reasons. First, errors normally occur in measurement because the accuracy of simple RF measuring equipment is relatively poor—even high-quality instruments suffer in accuracy compared with their low-frequency and dc counterparts. And second, the accuracy depends considerably on conditions—the antenna site, including height, terrain characteristics, and surroundings—under which the measurements are made. Calculations are frequently based on the measured or theoretical directive patterns of the antenna (see Chapter 2). The theoretical gain of an array may be determined approximately from:

$$G = 10 \log \frac{41{,}253}{\theta_H \, \theta_V} \qquad \text{(Eq 1)}$$

where

G = decibel gain over a dipole in its favored direction

$_H$ = horizontal half-power beamwidth in degrees

$_V$ = vertical half-power beamwidth in degrees.

This equation, strictly speaking, applies only to lossless antennas having approximately equal and narrow E- and H-plane beam widths—up to about 20°—and no large minor lobes. The E and H planes are discussed in Chapter 2. The error may be considerable when the formula is applied to simple directive antennas having relatively large beam widths. The error is in the direction of making the calculated gain larger than the actual gain.

Front-to-back ratio (F/B) is the ratio of the power radiated in the favored direction to the power radiated in the opposite direction. See Chapter 11 for a discussion of front-to-back ratio, and its close cousin, *worst-case front-to-rear ratio*.

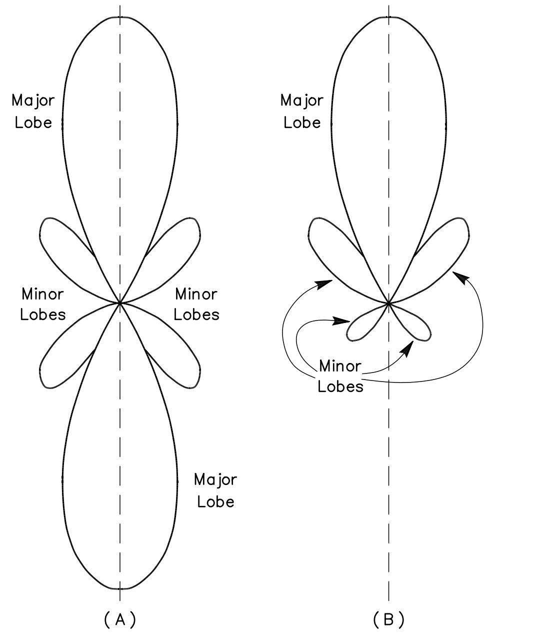


Fig 5—At A, typical bidirectional pattern and at B, unidirectional directive pattern. These drawings also illustrate the application of the terms *major* and *minor* to the pattern lobes.

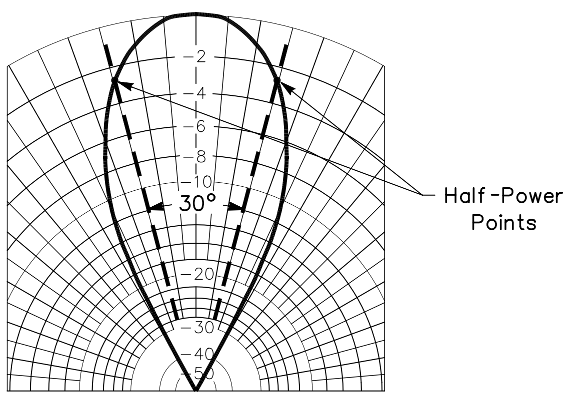


Fig 6—The width of a beam is the angular distance between the directions at which the received or transmitted power is half the maximum power (–3 dB). Each angular division of the pattern grid is 5°.

Phase

The term *phase* has the same meaning when used in connection with the currents flowing in antenna elements as it does in ordinary circuit work. For example, two currents are in phase when they reach their maximum values, flowing in the same direction, at the same instant. The direction of current flow depends on the way in which power is applied to the element.

This is illustrated in **Fig 7**. Assume that by some means an identical voltage is applied to each of the elements at the ends marked A. Assume also that the coupling between the elements is negligible, and that the instantaneous polarity of the voltage is such that the current is flowing away from the point at which the voltage is applied. The arrows show the assumed current directions. Then the currents in elements 1 and 2 are in phase, since they are flowing in the same direction in space and are caused by the same voltage. However, the current in element 3 is flowing in the *opposite* direction in space because the voltage is applied to the opposite end of the element. The current in element 3 is therefore 180° out of phase with the currents in elements 1 and 2.

The phasing of driven elements depends on the direc-

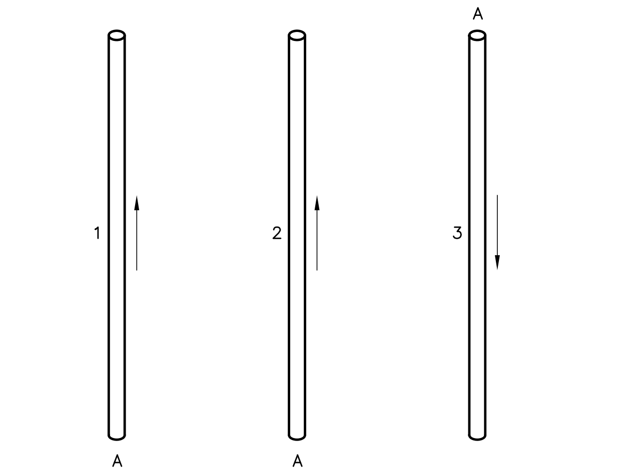


Fig 7—This drawing illustrates the phase of currents in antenna elements, represented by the arrows. The currents in elements 1 and 2 are in phase, while that in element 3 is 180° out of phase with 1 and 2.

tion of the element, the phase of the applied voltage, and the point at which the voltage is applied. In many systems used by amateurs, the voltages applied to the elements are exactly in or exactly out of phase with each other. Also, the axes of the elements are nearly always in the same direction, since parallel or collinear elements are invariably used. The currents in driven elements in such systems therefore are usually either exactly in or exactly out of phase with the currents in other elements.

It is possible to use phase differences of less than 180° in driven arrays. One important case is where the current in one set of elements differs by 90° from the current in another set. However, making provision for proper phasing in such systems is considerably more complex than in the case of simple 0° or 180° phasing, as described in a later section of this chapter.

In parasitic arrays the phase of the currents in the parasitic elements depends on the spacing and tuning, as described later.

Ground Effects

The effect of the ground is the same with a directive antenna as it is with a simple dipole antenna. The reflection factors discussed in Chapter 3 may therefore be applied to the vertical pattern of an array, subject to the same modifications mentioned in that chapter. In cases where the array elements are not all at the same height, the reflection factor for the mean height of the array may be used for a close approximation. The mean height is the average of the heights measured from the ground to the centers of the lowest and highest elements.

MUTUAL IMPEDANCE

Consider two $\lambda/2$ elements that are fairly close to each other. Assume that power is applied to only one element, causing current to flow. This creates an electromagnetic field, which induces a voltage in the second element and causes current to flow in it as well. The current flowing in element 2 will in turn induce a voltage in element 1, causing additional current to flow there. The total current in 1 is then the sum (taking phase into account) of the original current and the induced current.

With element 2 present, the amplitude and phase of the resulting current in element 1 will be different than if element 2 were not there. This indicates that the presence of the second element has changed the impedance of the first. This effect is called *mutual coupling*. Mutual coupling results in a *mutual impedance* between the two elements. The mutual impedance has both resistive and reactive components. The actual impedance of an antenna element is the sum of its self-impedance (the impedance with no other antennas present) and its mutual impedances with all other antennas in the vicinity.

The magnitude and nature of the feed-point impedance of the first antenna depends on the amplitude of the current induced in it by the second, and on the phase relationship between the original and induced currents. The amplitude and phase of the induced current depend on the spacing between the antennas and whether or not the second antenna is tuned to resonance.

In the discussion of the several preceding paragraphs, power is applied to only one of the two elements. Do not interpret this to mean that mutual coupling exists only in parasitic arrays! It is important to remember that mutual coupling exists between any two conductors that are located near one another.

Amplitude of Induced Current

The induced current will be largest when the two antennas are close together and are parallel. Under these conditions the voltage induced in the second antenna by the first, and in the first by the second, has its greatest value and causes the largest current flow. The coupling decreases as the parallel antennas are moved farther apart.

The coupling between collinear antennas is comparatively small, and so the mutual impedance between such antennas is likewise small. It is not negligible, however.

Phase Relationships

When the separation between two antennas is an appreciable fraction of a wavelength, a measurable period of time elapses before the field from antenna 1 reaches antenna 2. There is a similar time lapse before the field set up by the current in number 2 gets back to induce a current in number 1. Hence the current induced in antenna 1 by antenna 2 will have a phase relationship with the original current in antenna 1 that depends on the spacing between the two antennas.

The induced current can range all the way from being completely in phase with the original current to being completely out of phase with it. If the currents are in phase, the total current is larger than the original current, and the antenna feed-point impedance is reduced. If the currents are out of phase, the total current is smaller and the impedance is increased. At intermediate phase relationships the impedance will be lowered or raised depending on whether the induced current is mostly in or mostly out of phase with the original current.

Except in the special cases when the induced current is exactly in or out of phase with the original current, the induced current causes the phase of the total current to shift with respect to the applied voltage. Consequently, the presence of a second antenna nearby may cause the impedance of an antenna to be reactive—that is, the antenna will be detuned from resonance—even though its self-impedance is entirely resistive. The amount of detuning depends on the magnitude and phase of the induced current.

Tuning Conditions

A third factor that affects the impedance of antenna 1 when antenna 2 is present is the tuning of number 2. If antenna 2 is not exactly resonant, the current that flows in it as a result of the induced voltage will either lead or lag the phase it would have if the antenna were resonant. This causes an additional phase advance or delay that affects the phase of the current induced back in antenna 1. Such a phase lag has an effect similar to a change in the spacing between self-resonant antennas. However, a change in tuning is not exactly equivalent to a change in spacing because the two methods do not have the same effect on the amplitude of the induced current.

MUTUAL IMPEDANCE AND GAIN

The mutual coupling between antennas is important because it can have a significant effect on the amount of current that will flow for a given amount of power supplied. And it is the amount of *current* flowing that determines the field strength from the antenna. Other things being equal, if the mutual coupling between two antennas is such that the currents are greater for the same total power than would be the case if the two antennas were not coupled, the power gain will be greater than that shown in Table 1. On the other hand, if the mutual coupling is such as to reduce the current, the gain will be less than if the antennas were not coupled. The term *mutual coupling*, as used in this paragraph, assumes that the mutual impedance between elements is taken into account, along with the added effects of propagation delay because of element spacing, and element tuning or phasing.

The calculation of mutual impedance between antennas is a complex problem. Data for two simple but important cases are graphed in Figs 8 and 9. These graphs do not show the mutual impedance, but instead show a more useful quantity—the feed-point resistance measured at the center of an antenna as it is affected by the spacing between two antennas.

As shown by the solid curve in **Fig 8**, the feed-point resistance at the center of either antenna, when the two are self-resonant, parallel, and operated in phase, decreases as

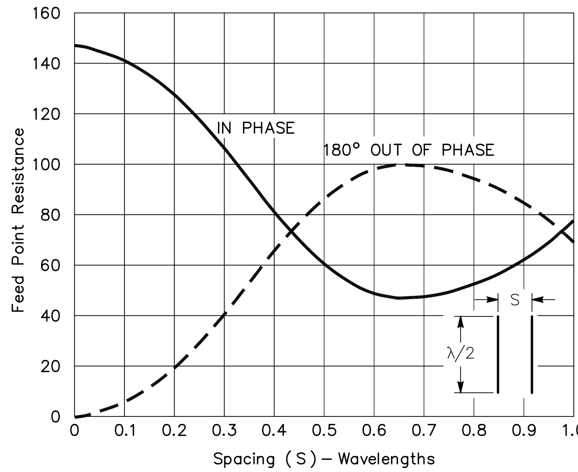


Fig 8—Feed-point resistance measured at the center of one element as a function of the spacing between two parallel $^1/_2$-λ self-resonant antenna elements. For ground-mounted $^1/_4$-λ vertical elements, divide these resistances by two.

the spacing between them is increased until the spacing is about 0.7 λ. This is a broadside array. The maximum gain is achieved from a pair of such elements when the spacing is in this region, because the current is larger for the same power and the fields from the two arrive in phase at a distant point placed on a line perpendicular to the line joining the two antennas.

The dashed line in Fig 8, representing two antennas operated 180° out of phase (end-fire), cannot be interpreted quite so simply. The feed-point resistance decreases with spacing decreasing less than about 0.6 λ in this case. However, for the range of spacings considered, only when the spacing is 0.5 λ do the fields from the two antennas add up exactly in phase at a distant point in the favored direction. At smaller spacings the fields become increasingly out of

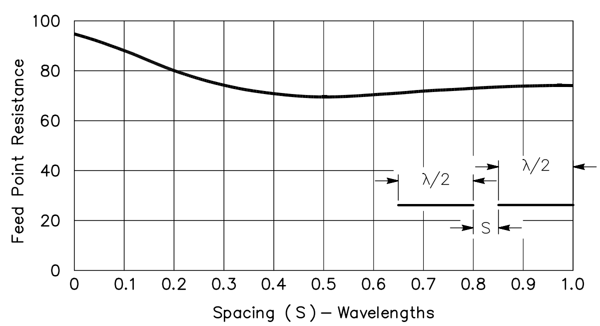


Fig 9—Feed-point resistance measured at the center of one element as a function of the spacing between the ends of two collinear self-resonant $^1/_2$-λ antenna elements operated in phase.

phase, so the total field is less than the simple sum of the two. Smaller spacings thus decrease the gain at the same time that the reduction in feed point resistance is increasing it. For a lossless antenna, the gain goes through a maximum when the spacing is in the region of $^1/_8$ λ.

The feed-point resistance curve for two collinear elements in phase, **Fig 9**, shows that the feed-point resistance decreases and goes through a broad minimum in the region of 0.4 to 0.6-λ spacing between the adjacent ends of the antennas. As the minimum is not significantly less than the feed-point resistance of an isolated antenna, the gain will not exceed the gain calculated on the basis of uncoupled antennas. That is, the best that two collinear elements will give, even with optimum spacing, is a power gain of about 2 (3 dB). When the separation between the ends is very small—the usual method of operation—the gain is reduced.

GAIN AND ARRAY DIMENSIONS

The gain of an array is principally determined by the dimensions of the array as long as there are a minimum number of elements. A good example of this is the relationship between boom length, gain and number of elements for an array such as a Yagi. **Fig 10** compares the gain versus boom length for Yagis with different numbers of elements. Notice that, for given number of elements, the gain increases as the boom length increases, up to a maximum. Beyond this point, longer boom lengths result in less gain for a given number of elements. This observation does not mean that it is always desirable to use only the minimum number of elements. Other considerations of array performance, such as front-to-back ratio, minor lobe amplitudes or operating bandwidth, may make it advantageous to use more than the minimum number of elements for a given array length. A specific example of this is presented in a later section in a comparison between a half-square, a bobtail curtain and a Bruce array.

In a broadside array the gain is a function of both the length and width of the array. The gain can be increased by adding more elements (with additional spacing) or by using longer elements (>λ/2), although the use of longer elements requires proper attention to current phase in the elements. In general, in a broadside array the element spacing that gives maximum gain for a minimum number of elements is in the range of 0.5 to 0.7 λ. Broadside arrays with elements spaced for maximum gain will frequently have significant side lobes and associated narrowing of the main lobe beamwidth. Side lobes can be reduced by using more than the minimum number of elements, spaced closer than the maximum gain distance.

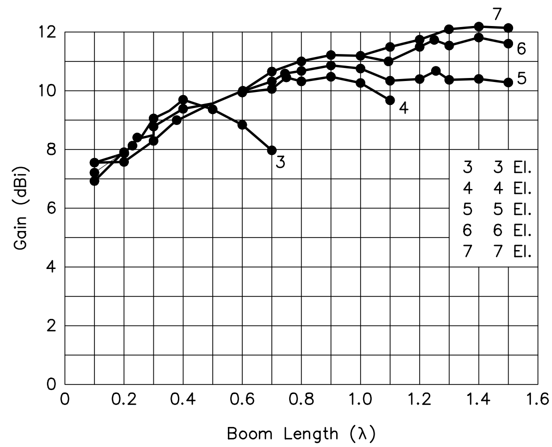


Fig 10—Yagi gain for 3, 4, 5, 6 and 7-element beams as a function of boom length. (From *Yagi Antenna Design*, J. Lawson, W2PV.)

Additional gain can be obtained by expanding the array into a third dimension. An example of this is the stacking of endfire arrays in a broadside configuration. In the case of stacked short endfire arrays, maximum gain occurs with spacings in the region of 0.5 to 0.7 λ. However, for longer higher-gain end-fire arrays, larger spacing is required to achieve maximum gain. This is important in VHF and UHF arrays, which often use long-boom Yagis.

PARASITIC ARRAYS

The foregoing applies to multi-element arrays of both types, driven and parasitic. However, there are special considerations for driven arrays that do not necessarily apply to parasitic arrays, and vice versa. Such considerations for Yagi and quad parasitic arrays are presented in Chapters 11 and 12. The remainder of this chapter is devoted to driven arrays.

Driven Arrays

Driven arrays in general are either broadside or endfire, and may consist of collinear elements, parallel elements, or a combination of both. From a practical standpoint, the maximum number of usable elements depends on the frequency and the space available for the antenna. Fairly elaborate arrays, using as many as 16 or even 32 elements, can be installed in a rather small space when the operating frequency is in the VHF range, and more at UHF. At lower frequencies the construction of antennas with a large number of elements is impractical for most amateurs.

Of course the simplest of driven arrays is one with just two elements. If the elements are collinear, they are always fed in phase. The effects of mutual coupling are not great, as illustrated in Fig 9. Therefore, feeding power to each element in the presence of the other presents no significant problems. This may not be the case when the elements are parallel to each other. However, because the combination of spacing and phasing arrangements for parallel elements is infinite, the number of possible radiation patterns is endless. This is illustrated in **Fig 11**. When the elements are fed in phase, a broadside pattern always results. At spacings of less than $^5/_8$ λ with the elements fed 180° out of phase, an end-fire pattern always results. With intermediate amounts of phase difference, the results cannot be so simply stated. Patterns evolve that are not symmetrical in all four quadrants.

Because of the effects of mutual coupling between the two driven elements, for a given power input greater or lesser currents will flow in each element with changes in spacing and phasing, as described earlier. This, in turn, affects the gain of the array in a way that cannot be shown merely by plotting the *shapes* of the patterns, as has been done in Fig 11. Therefore, supplemental gain information is also shown in Fig 11, adjacent to the pattern plot for each combination of spacing and phasing. The gain figures shown are referenced to a single element. For example, a pair of elements fed 90° apart at a spacing of $^1/_4$ λ will have a gain in the direction of maximum radiation of 3.1 dB over a single element.

Current Distribution in Phased Arrays

In the plots of Fig 11, the two elements are assumed to be identical and self-resonant. In addition, currents of equal amplitude are assumed to be flowing at the feed point of each element, a condition that most often will not exist in practice without devoting special consideration to the feeder system. Such considerations are discussed in the next section of this chapter.

Most literature for radio amateurs concerning phased arrays is based on the assumption that if all elements in the array are identical, the *current distribution* in all the elements will be identical. This distribution is presumed to be that of a single, isolated element, or nearly sinusoidal. However, information published in the professional literature as early as the 1940s indicates the existence of dissimilar current distributions among the elements of phased arrays. (See Harrison and King references in the Bibliography.) Lewallen, in July 1990 *QST*, points out the causes and effects of dissimilar current distributions.

In essence, even though the two elements in a phased array may be identical and have exactly equal currents of the desired phase flowing *at the feed point*, the amplitude and phase relationships degenerate with departure from the feed point. This happens any time the phase relationship is not 0° or 180°. Thus, the field strengths produced at a distant point by the individual elements may differ. This is because the field from each element is determined by the *distribution* of the current, as well as its magnitude and phase. The effects are minimal with shortened elements—verticals less than $^1/_4$ λ or dipoles less than $^1/_2$ λ long. The effects on radiation patterns begin to show at the above resonant lengths, and become profound with longer elements— $^1/_2$ λ or longer verticals and 1 λ or longer center-fed elements. These effects are less pronounced with thin elements. The amplitude and phase degeneration takes place because the currents in the array elements are not sinusoidal. Even in two-element arrays with phasing of 0° or 180°, the currents are not sinusoidal, but in these two special cases they do remain identical.

The pattern plots of Fig 11 take element current distributions into account. The visible results of dissimilar distributions are incomplete nulls in some patterns, and the development of very small minor lobes in others. For example, the pattern for a phased array with 90° spacing and 90° phasing has traditionally been published in amateur literature as a cardioid with a perfect null in the rear direction. Fig 11, calculated for 7.15-MHz self-resonant dipoles of #12 wire in free space, shows a minor lobe at the rear and only a 33-dB front-to-back ratio.

It is characteristic of broadside arrays that the power gain is proportional to the length of the array but is substantially independent of the number of elements used, provided the optimum element spacing is not exceeded. This means, for example, that a five-element array and a six-element array will have the same gain, provided the elements in both are spaced so the overall array length is the same. Although this principle is seldom used for the purpose of reducing the number of elements because of complications introduced in feeding power to each element in the proper phase, it does illustrate the fact that there is nothing to be gained, in terms of more gain, by increasing the number of elements if the space occupied by the antenna is not increased proportionally.

Generally speaking, the maximum gain in the smallest linear dimensions will result when the antenna combines both broadside and end-fire directivity and uses both parallel and collinear elements. In this way the antenna is spread over a greater volume of space, which has the same effect as extending its length to a much greater extent in one linear direction.

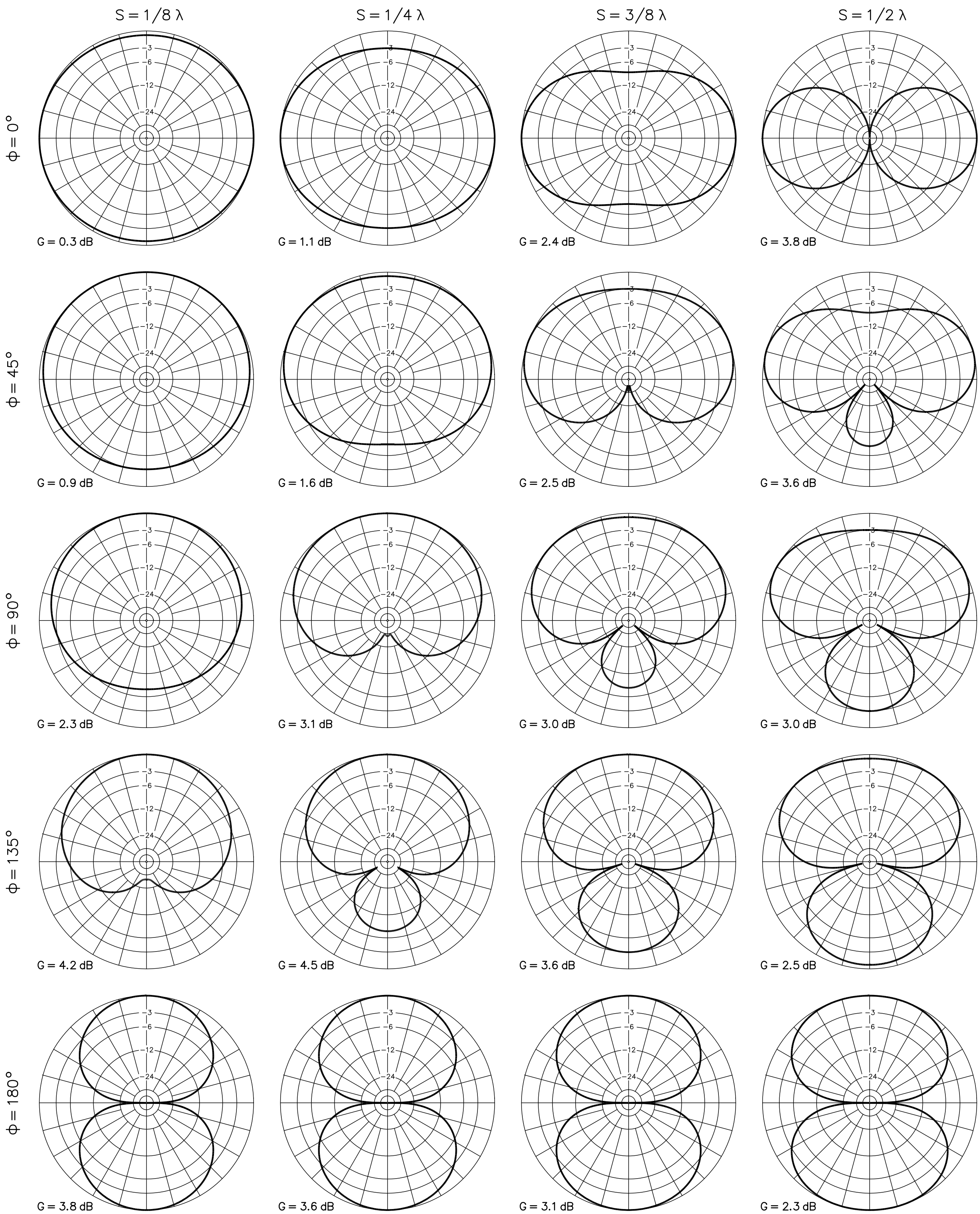
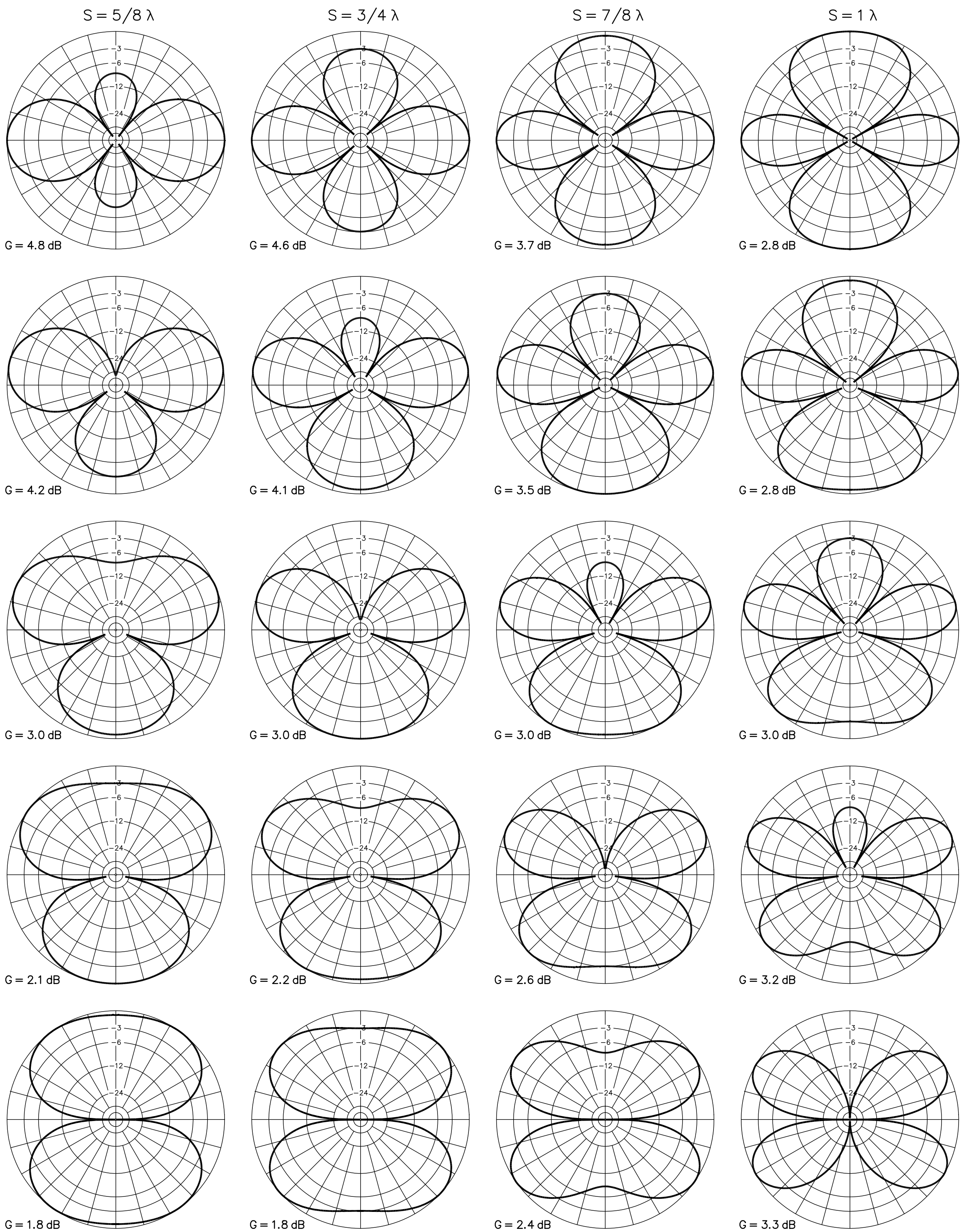


Fig 11—H-plane patterns of two identical parallel driven elements, spaced and phased as indicated (S = spacing, φ = phasing). The elements are aligned with the vertical (0°-180°) axis, and the element nearer the 0° direction (top of page) is of lagging phase at angles other than 0°. The two elements are assumed to be thin and self-resonant, with equal-amplitude currents flowing at the feed point. See text regarding current distributions. The gain figure



associated with each pattern indicates that of the array over a single element. The plots represent the horizontal or azimuth pattern at a 0° elevation angle of two 1/4-λ vertical elements over a perfect conductor, or the free-space vertical or elevation pattern of two horizontal 1/2-λ elements when viewed on end, with one element above the other. (Patterns computed with *ELNEC*—see Bibliography.)

Phased Array Techniques

Phased antenna arrays have become increasingly popular for amateur use, particularly on the lower frequency bands, where they provide one of the few practical methods of obtaining substantial gain and directivity. This section on phased array techniques was written by Roy W. Lewallen, W7EL. The operation and limitations of phased arrays, how to design feed systems to make them work properly, and how to make necessary tests and adjustments are discussed in the pages that follow. The examples deal primarily with vertical HF arrays, but the principles apply to horizontal and VHF/UHF arrays as well.

The performance of a phased array is determined by several factors. Most significant among these are the characteristics of a single element, reinforcement or cancellation of the fields from the elements, and the effects of mutual coupling. To understand the operation of phased arrays, it is first necessary to understand the operation of a single antenna element.

Fundamentals of Phased Arrays

Of primary importance is the strength of the field produced by the element. The field radiated from a linear (straight) element, such as a dipole or vertical monopole, is proportional to the sum of the elementary currents flowing in each part of the antenna element. For this discussion it is important to understand what determines the current in a single element.

The amount of current flowing at the base of a resonant ground mounted vertical or ground-plane antenna is given by the familiar formula

$$I = \sqrt{\frac{P}{R}} \qquad \text{(Eq 2)}$$

where

P is the power supplied to the antenna

R is the feed-point resistance.

R consists of two parts, the loss resistance and the radiation resistance. The loss resistance, R_L, includes losses in the conductor, in the matching and loading components, and dominantly (in the case of ground-mounted verticals), in ground losses. The power *dissipated* in the radiation resistance, R_R, is the power that is radiated, so maximizing the power dissipated by the radiation resistance is desirable. However, the power dissipated in the loss resistance truly is lost (as heat), so resistive losses should be made as small as possible.

The radiation resistance of an element may be derived from electromagnetic field theory, being a function of antenna length, diameter, and geometry. Graphs of radiation resistance versus antenna length are given in Chapter 2. The radiation resistance of a thin $^1/_4$-λ ground-mounted vertical is about 36 Ω. A $^1/_2$-λ dipole in free space has a radiation resistance of about 73 Ω. Reducing the antenna lengths by one half drops the radiation resistances to approximately 7 and 14 Ω, respectively.

Radiation Efficiency

To generate a stronger field from a given radiator, it is necessary either to increase the power P (the brute-force solution), or to decrease the loss resistance R_L (by putting in a more elaborate ground system for a vertical, for instance), or to somehow decrease the radiation resistance R_R so more current will flow with a given power input. This can be seen by expanding the formula for base current as:

$$I = \sqrt{\frac{P}{R_R + R_L}} \qquad \text{(Eq 3)}$$

Splitting the feed-point resistance into components R_R and R_L easily leads to an understanding of element efficiency. The efficiency of an element is the proportion of the total power that is actually radiated. The roles of R_R and R_L in determining efficiency can be seen by analyzing a simple equivalent circuit, shown in **Fig 12**.

The power dissipated in R_R (the radiated power) equals I^2R_R. The total power supplied to the antenna system is

$$P = I^2 (R_R + R_L) \qquad \text{(Eq 4)}$$

so the efficiency (the fraction of supplied power that is actually radiated) is

$$\text{Eff} = \frac{I^2 R_R}{I^2 (R_R + R_L)} = \frac{R_R}{R_R + R_L} \qquad \text{(Eq 5)}$$

Efficiency is frequently expressed in percent, but expressing it in decibels relative to a 100%-efficient radiator gives a better idea of what to expect in the way of signal strength. The field strength of an element relative to a lossless but otherwise identical element, in dB, is

$$\text{FSG} = 10 \log \frac{R_R}{R_R + R_L} \qquad \text{(Eq 6)}$$

where FSG = field strength gain, dB.

For example, information presented by Sevick in March 1973 *QST* shows that a $^1/_4$-λ ground-mounted vertical antenna with four 0.2-λ radials has a feed-point resistance of about 65 Ω (see the Bibliography at the end of this chapter). The efficiency of such a system is 36/65 = 55.4%. It is rather disheartening to think that, of 100 W fed to the antenna, only 55 W are being radiated, with the remainder literally warming up the ground. Yet the signal will be only 10 log (36/65) = –2.57 dB relative to the same vertical with a perfect ground system. In view of this information, trading a small reduction in signal strength for lower cost and greater simplicity may become an attractive consideration.

So far, only the current at the base of a resonant antenna has been discussed, but the field is proportional to

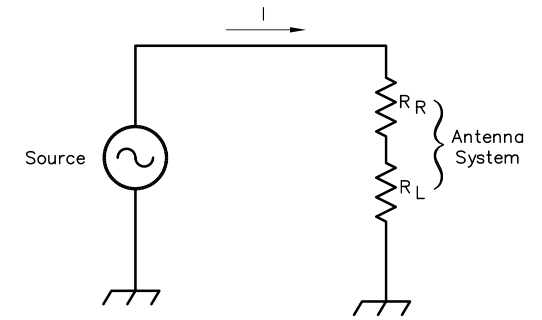


Fig 12—Simplified equivalent circuit for a single-element resonant antenna. R_R represents the radiation resistance, and R_L the ohmic losses in the total antenna system.

the sum of currents in each tiny part of the antenna. The field is a function of not only the magnitude of current flowing at the base, but also the distribution of current along the radiator and the length of the radiator. However, nothing can be done at the base of the antenna to change the current distribution, so for a given element, the field strength is proportional to the base current (or center current, in the case of a dipole). However, changing the radiator length or loading it at some point other than the feed point will change the current distribution. More information on shortened or loaded radiators may be found in Chapters 2 and 6, and in the Bibliography references of this chapter. A few other important facts follow.

1) If there is no loss, the field from even an infinitesimally short radiator is less than $^1/_2$ dB weaker than the field from a half-wave dipole or quarter-wave vertical. Without loss, all the supplied power is radiated regardless of the antenna length, so the only factor influencing gain is the slight difference in the patterns of very short and $^1/_2$-λ antennas. The small pattern difference arises from different current distributions. A short antenna has a very low radiation resistance, resulting in a heavy current flow over its short length. In the absence of loss, this generates a field strength comparable to that of a longer antenna. Where loss is present—that is, in practical antennas—shorter radiators usually don't do so well, since the low radiation resistance leads to lower efficiency for a given loss resistance. If care is taken, short antennas can achieve good efficiency.
2) The feed-point resistance of folded antennas isn't the radiation resistance as the term is used here. The act of folding an antenna only transforms the input impedance to a higher value, providing an easier match in some cases. The higher feed-point impedance doesn't help the efficiency, since the resulting smaller currents flow through more conductors, for the same net loss. In a folded vertical, the same total current ends up flowing through the ground system, again resulting in the same loss.
3) The current flowing in an element with a given power input can be increased, or decreased, by mutual coupling to other elements. The effect is equivalent to changing the element radiation resistance. Mutual coupling is sometimes regarded as a *minor* effect, but most often it is not minor!

Field Reinforcement and Cancellation

Consider two elements that each produce a field strength of, say, exactly 1 millivolt per meter (mV/m) at some distance many wavelengths from the array. In the direction in which the fields are in phase, a total field of 2 mV/m results; in the direction in which they are out of phase, a zero field results. The ratio of maximum to minimum field strength of this array is 2/0, or infinite.

Now suppose, instead, that one field is 10% high and the other 10% low—1.1 and 0.9 mV/m, respectively. In the forward direction, the field strength is still 2 mV/m, but in the canceling direction, the field will be 0.2 mV/m. The front-to-back ratio has dropped from infinite to 2/0.2, or 20 dB. (Actually, slightly more power is required to redistribute the field strengths this way, so the forward gain is reduced—but only by a small amount, less than 0.1 dB.) For most arrays, unequal fields from the elements have a minor effect on forward gain, but a major effect on pattern nulls.

Even with perfect current balance, deep nulls aren't assured. **Fig 13** shows the minimum spacing required for total field reinforcement or cancellation. If the element spacing isn't adequate, there may not be any direction in which the fields are completely out of phase (see curve B of Fig 13). Slight physical and environmental differences between elements will invariably affect null depths, and null depths will vary with elevation angle. However, a properly designed and fed array can, in practice, produce very impressive nulls. The key to achieving good performance is being able to control the fields from the elements. This, in turn, requires knowing how to control the currents in the

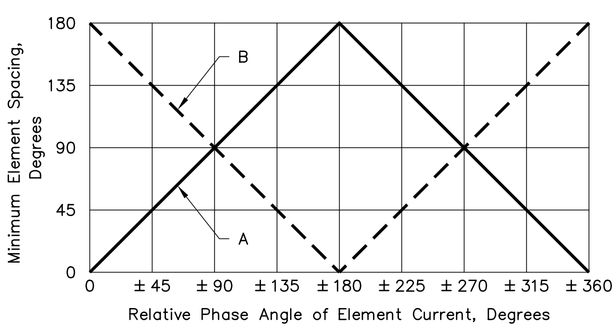


Fig 13—Minimum element spacing required for total field reinforcement, curve A, or total field cancellation, curve B. Total cancellation results in pattern nulls in one or more directions. Total reinforcement does not necessarily mean there is gain over a single element, as the effects of loss and mutual coupling must also be considered.

elements, since the fields are proportional to the currents. Most phased arrays require the element currents to be equal in magnitude and different in phase by some specific amount. Just how this can be accomplished is explained in a subsequent section.

MUTUAL COUPLING

Mutual coupling refers to the effects which the elements in an array have on each other. Mutual coupling can occur intentionally or entirely unintentionally. For example, Lewallen has observed effects such as a quad coupling to an inverted-V dipole to form a single, very strange, antenna system. The current in the "parasitic element" (nondriven antenna) was caused entirely by mutual coupling, just as in the familiar Yagi antenna. The effects of mutual coupling are present regardless of whether or not the elements are driven.

Suppose that two driven elements are very far from each other. Each has some voltage and current at its feed point. For each element, the ratio of this voltage to current is the element self-impedance. If the elements are brought close to each other, the current in each element will change in amplitude and phase because of coupling with the field from the other element. Significant mutual coupling occurs at spacings as great as a wavelength or more. The fields change the currents, which change the fields. There is an equilibrium condition where the currents in all elements (hence, their fields) are totally interdependent. The feed-point impedances of all elements also are changed from their values when far apart, and all are dependent on each other. In a driven array, the changes in feed-point impedances can cause additional changes in element currents, because the operation of many feed systems depends on the element feed-point impedances.

Connecting the elements to a feed system to form a driven array does not eliminate the effects of mutual coupling. In fact, in many driven arrays the mutual coupling has a greater effect on antenna operation than the feed system does. All feed-system designs must account for the impedance changes caused by mutual coupling if the desired current balance and phasing are to be achieved.

Several general statements can be made regarding phased-array systems. Mutual coupling accounts for these characteristics.

1) The resistances and reactances of all elements of an array generally will change substantially from the values of an isolated element.
2) If the elements of a two-element array are identical and have equal currents, which are in phase or 180° out of phase, the feed-point impedances of the two elements will be equal. But they will be different than for an isolated element. If the two elements are part of a larger array, their impedances can be very different from each other.
3) If the elements of a two-element array have currents that are neither in phase (0°) nor out of phase (180°), their feed-point impedances will not be equal. The difference will be substantial in typical amateur arrays.
4) The feed-point resistances of the elements in a closely spaced, 180° out-of-phase array will be very low, resulting in poor efficiency unless care is taken to minimize loss. This is also true for any other closely spaced array with significant predicted gain.

Gain

Gain is strictly a relative measure, so the term is completely meaningless unless accompanied by a statement of just what it is relative to. One useful measure for phased array gain is *gain relative to a single similar element*. This is the increase in signal strength that would be obtained by replacing a single element by an array made from elements just like it. All gain figures in this section are relative to a single similar element unless otherwise noted. In some instances, such as investigating what happens to array performance when *all* elements become more lossy, gain refers to a more absolute, although unattainable standard; a lossless element. Uses of this standard are explicitly noted.

Why does a phased array have gain? One way to view it is in terms of directivity. Since a given amount of radiated power, whether radiated from one or a dozen elements, must be radiated *somewhere*, field strength must be increased in some directions if it is reduced in others. There is no guarantee that the fields from the elements of an arbitrary array will completely reinforce or cancel in any direction; element spacing must be adequate for either to happen (see Fig 13). If the fields reinforce or cancel to only a single extent, causing a pattern similar to that of a single element, the gain will also be similar to that of a single element.

To get a feel for how much gain a phased array can deliver, consider what would happen if there were no change in element feed-point resistance from mutual coupling. This actually does occur at some spacings and phasings, but not in commonly used systems. It is a useful example, nevertheless.

In the fictitious array the elements are identical and there are no resistance changes from mutual coupling. The feed-point resistance, R_F, equals $R_R + R_L$, the sum of radiation and loss resistances. If power P is put into a single element, the feed-point current is

$$I_F = \sqrt{\frac{P}{R_F}} \qquad \text{(Eq 7)}$$

At a given distance, the field strength is proportional to the current, so the field strength is

$$E = kI_F = k\sqrt{\frac{P}{R_F}} \qquad \text{(Eq 8)}$$

where k is the constant relating the element current to the field strength at the chosen distance.

If, instead, the power is equally split between two elements,

$$I_{F1}=I_{F2}=\sqrt{\frac{P/2}{R_F}} \quad \text{(Eq 9)}$$

From this,

$$E_1=E_2=k\sqrt{\frac{P/2}{R_F}} \quad \text{(Eq 10)}$$

If the elements are spaced far enough apart to allow full field reinforcement, the total field in the favored direction will be

$$E_1=E_2=2k\sqrt{\frac{P/2}{R_F}}=\sqrt{2}\,k\sqrt{\frac{P}{R_F}} \quad \text{(Eq 11)}$$

This represents a field strength gain of

$$FSG=20\ \log\ \sqrt{2}=3\ dB \quad \text{(Eq 12)}$$

where FSG = field strength gain, dB.

The power gain in dB equals the field strength gain in dB. The above argument leading to Eq 11 can be extended to show that the gain in dB for an array of n elements, without resistance changes from mutual coupling and with sufficient spacing and geometry for total field reinforcement, is

$$FSG=20\ \log\ \sqrt{n}=10\ \log\ n \quad \text{(Eq 13)}$$

That is, a five-element array satisfying these assumptions would have a power gain of 5 times, or about 7 dB. Remember, the assumption was made that equal power is fed to each element. With equal element resistances and no resistance changes from mutual coupling, *equal currents* are therefore made to flow in all elements.

The gain of an array can be increased or decreased from 10 log n decibels by mutual coupling, but any loss will move the gain back toward 10 log n. This is because resistance changes from mutual coupling get increasingly swamped by the loss as the loss increases. Arrays designed to have substantially more gain than 10 log n decibels require heavy element currents. As designed gain increases, the required currents increase dramatically, resulting in power losses that partially or totally negate the expected gain. The net result is a practical limit of about 10 log n for the gain in dB of an n-element array, and this gain can be achieved only if extreme attention is paid to keeping losses very small. The majority of practical arrays, particularly arrays of ground-mounted verticals, have gains closer to 10 log n decibels.

The foregoing comments indicate that many of the claims about the gain of various arrays are exaggerated, if not ridiculous. But an honest 3 dB or so of gain from a two-element array can really be appreciated if an equally honest 3 dB has been attempted by other means. Equations for calculating array gain and examples of their use are given in a later section of this chapter.

FEEDING PHASED ARRAYS

The previous section explains why the currents in the elements must be very close to the ratios required by the array design. This section explains how to feed phased arrays to produce the desired current ratio and phasing. Since the desired current ratio is 1:1 for virtually all two-element and for most larger amateur arrays, special attention is paid to methods of assuring equal element currents. Other current ratios are also examined.

Phasing Errors

For an array to produce the desired pattern, the element currents must have the required magnitude and the required phase relationship. On the surface, this sounds easy; just make sure that the difference in electrical lengths of the feed lines to the elements equals the desired phase angle. Unfortunately, this approach doesn't necessarily achieve the desired result. The first problem is that the phase shift through the line is not equal to its electrical length. The current (or, for that matter, voltage) delay in a transmission line is equal to its electrical length in only a few special cases—cases which do not exist in most amateur arrays! The impedance of an element in an array is frequently very different from the impedance of an isolated element, and the impedances of all the elements in an array can be different from each other.

Consequently, the elements seldom provide a matched load for the element feed lines. The effect of mismatch on

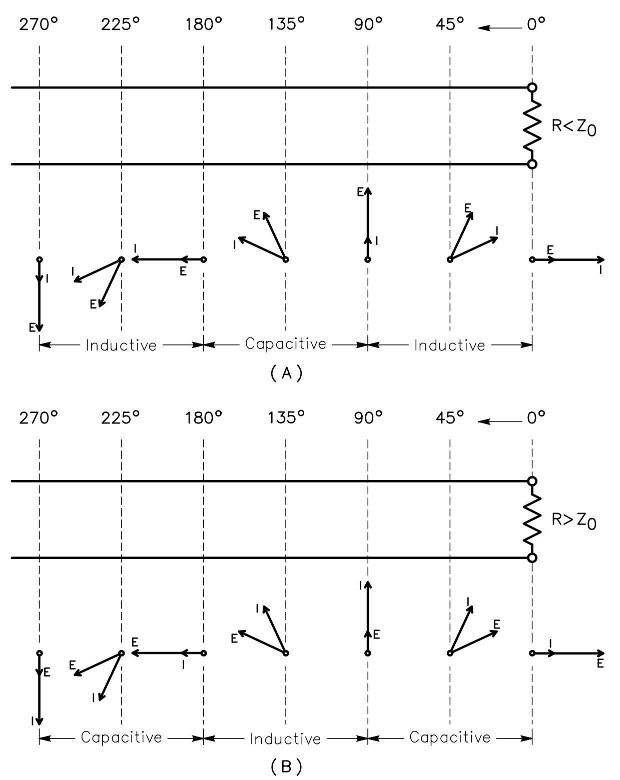


Fig 14—Resultant voltages and currents along a mismatched line. At A, R less than Z_0, and at B, R greater than Z_0.

phase shift can be seen in **Fig 14**. Observe what happens to the phase of the current and voltage on a line terminated by a purely resistive impedance which is lower than the characteristic impedance of the line (Fig 14A). At a point 45° from the load, the current has advanced less than 45°, and the voltage more than 45°. At 90° from the load, both are advanced 90°. At 135°, the current has advanced more and the voltage less than 135°. This apparent slowing down and speeding up of the current and voltage waves is caused by interference between the forward and reflected waves. It occurs on any line not terminated with a pure resistance equal to its characteristic impedance. If the load resistance is greater than the characteristic impedance of the line, as shown in Fig 14B, the voltage and current exchange angles. Adding reactance to the load causes additional phase shift. The *only* cases in which the current (or voltage) delay is equal to the electrical length of the line are

1) When the line is *flat*, that is, terminated in a purely resistive load equal to its characteristic impedance;
2) When the line length is an integral number of half wavelengths;
3) When the line length is an odd number of quarter wavelengths *and* the load is purely resistive; and
4) When other specific lengths are used for specific load impedances.

Just how much phase error can be expected if two lines are simply hooked up to form an array? There is no simple answer. Some casually designed feed systems might deliver satisfactory results, but most will not. Later examples show just what the consequences of casual feeding can be.

The effect of phasing errors is to alter the basic shape of the radiation pattern. Nulls may be reduced in depth, and additional lobes added. Actual patterns can be calculated by using Eq 15 in a later section of this chapter. The effects of phasing errors on the shape of a 90° fed, 90° spaced array pattern are shown in **Fig 15**.

A second problem with simply connecting feed lines of different lengths to the elements is that the lines will change the *magnitudes* of the currents. The magnitude of the current (or voltage) out of a line does not equal the magnitude in, except in cases 1, 2 and 4 above. The feed systems presented here assure currents which are correct in both magnitude and phase.

The Wilkinson Divider

The *Wilkinson divider*, sometimes called the *Wilkinson power divider*, has been promoted in recent years as a means to distribute power among the elements of a phased array. It is therefore worthwhile to investigate just what the Wilkinson divider does.

The Wilkinson divider is shown in **Fig 16**. It is a very useful device for *splitting power* among several loads, or, in reverse, combining the outputs from several generators. If all loads are equal to the design value (usually 50 Ω), the power from the source is split equally among them, and no power is dissipated in the resistors. If the impedance of one of the loads should change, however, the power which was being delivered to that load becomes shared between it and the resistors. The power to the other loads is unchanged, so they are not affected by the errant load.

The network is also commonly used to combine the outputs of several transmitters to obtain a higher power than a single transmitter can deliver. The great value of the network becomes evident by observing what happens if one transmitter fails. The other transmitters continue working normally, delivering their full power to the load. The Wilkinson network prevents them, or the load, from *seeing* the failed transmitter, except as a reduction of total output power. Most other combining techniques would result in incorrect operation or failure of the remaining transmitters.

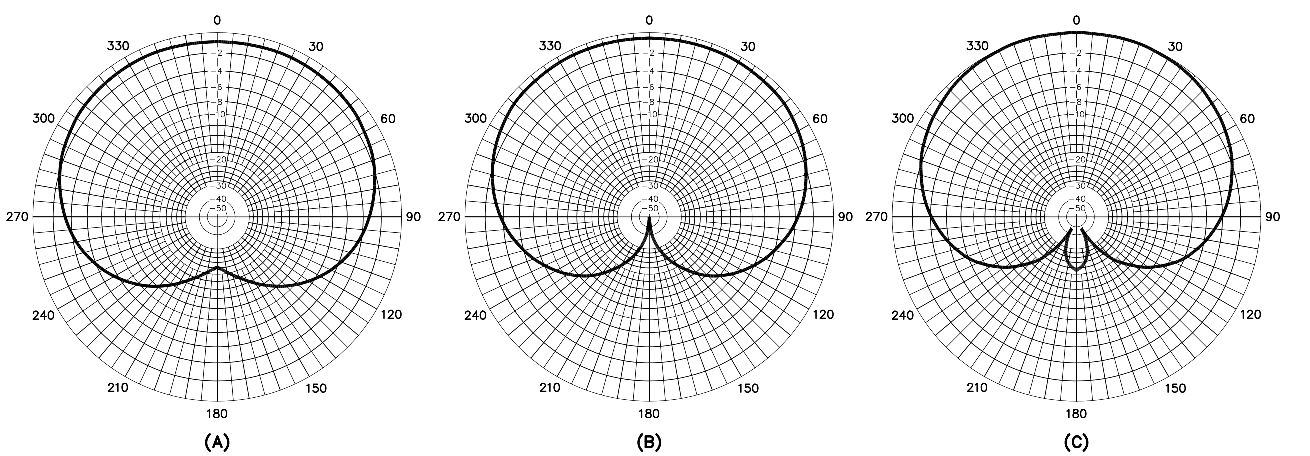


Fig 15—The change in pattern of a 90° spaced array caused by deviations from 90° phasing (equal currents and similar current distributions assumed). At A, B and C the respective phase angles are 80°, 90° and 100°. Note the minor changes in gain as well as in pattern shapes with phase angle deviations. Gain is referenced to a single element; add 3.4 dB to the scale values shown for each plot.

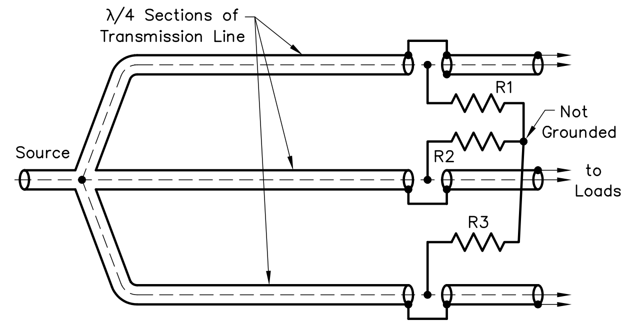


Fig 16—The Wilkinson divider. Three output ports are shown here, but the number may be reduced to two or increased as necessary. If (and only if) the source and all load impedances equal the design impedance, the power from the source will be split equally among the loads. The Z_0 of the $^1/_4$-λ sections is equal to the load impedance times the square root of the number of loads.
R1, R2, R3—Noninductive resistors having a value equal to the impedance of the loads.

The Wilkinson divider is a port-to-port isolation device. It does *not* assure equal powers or currents in all loads.When connected to a phased array, it might make the system more broadband—by an amount directly related to the amount of power being lost in the resistors! Amateurs feeding a *four-square* array (reference Atchley, Steinhelfer and White—see Bibliography) with this network have reported one or more resistors getting very warm, indicating lost power that would be used to advantage if radiated.

Incidentally, if the divider is to be used for its intended purpose, the source impedance must be correct for proper operation. Hayward and DeMaw have pointed out that amateur transmitters do not necessarily have a well-defined output impedance (see Bibliography).

In summary, if the Wilkinson divider is used for feeding a phased array, (1) it will *not* assure equal element powers (which are not wanted anyway). (2) It will *not* assure equal element currents (which *are* wanted). (3) It will waste power. The Wilkinson divider is an extremely useful device. But it is not what is needed for feeding phased antenna arrays.

The Broadcast Approach

Networks can be designed to transform the element base impedances to, say, 50 Ω resistive. Then another network can be inserted at the junction of the feed lines to properly divide the power among the elements (not necessarily equally!). And finally, additional networks must be built to correct for the phase shifts of the other networks. This general approach is used by the broadcast industry. Although this technique can be used to feed any type of array, design is difficult and adjustment is tedious, as all adjustments interact. When the relative currents and phasings are adjusted, the feed-point impedances change, which in turn affect the element currents and phasings, and so on. A further disadvantage of using this method is that switching the array direction is generally impossible. Information on applying this technique to amateur arrays may be found in Paul Lee's book.

A PREFERRED FEED METHOD

The feed method introduced here has been used in its simplest form to feed television receiving antennas and other arrays, as presented by Jasik, pages 2-12 and 24-10. However, this feed method has not been widely applied to amateur arrays.

The method takes advantage of an interesting property of $^1/_4$-λ transmission lines. (All references to lengths of lines are electrical length, and lines are assumed to have negligible loss.) See **Fig 17**. The magnitude of the *current* out of a $^1/_4$-λ transmission line is equal to the *input* voltage divided by the characteristic impedance of the line, independent of the load impedance. In addition, the phase of the output current lags the phase of the input voltage by 90°, also independent of the load impedance. This property can be used to advantage in feeding arrays with certain phasings between elements.

If any number of loads are connected to a common driving point through $^1/_4$-λ lines of equal impedance, the currents in the loads will be *forced* to be equal and in phase, regardless of the load impedances. So any number of in-phase elements can be correctly fed using this method. Arrays which require unequal currents can be fed through lines of unequal impedance to achieve other current ratios.

The properties of $^1/_2$-λ lines also are useful. Since the current out of a $^1/_2$-λ line equals the input current shifted 180°, regardless of the load impedance, any number of half wavelengths of line may be added to the basic $^1/_4$ λ, and the current and phase "forcing" property will be preserved. For example, if one element is fed through a $^1/_4$-λ line, and another element is fed from the same point through a $^3/_4$-λ line of the same characteristic impedance, the currents in the two elements will be forced to be equal in magnitude and 180° out of phase, regardless of the feed-point impedances of the elements.

If an array of two identical elements is fed in phase or 180° out of phase, both elements have the same feed-point

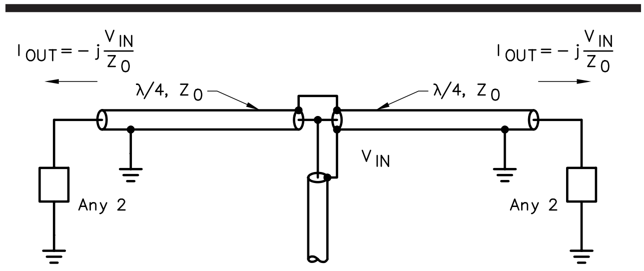


Fig 17—A useful property of $^1/_4$-λ transmission lines; see text. This property is utilized in the "current forcing" method of feeding an array of coupled elements.

impedance. With these arrays, feeding the elements through equal lengths of feed line (in phase) or lengths differing by 180° (out of phase) will lead to the correct current and phase match, regardless of what the line length is. Unless the lines are an integral number of half wavelengths long, the currents out of the lines will not be equal to the input currents, and the phase will not be shifted an amount equal to the electrical lengths of the lines. But both lines will produce the same transformation and phase shift because their load impedances are equal, resulting in a properly fed array. In practice, however, feed-point impedances of elements frequently are different even in these arrays, because of such things as different ground systems (for vertical elements), proximity to buildings or other antennas, or different heights above ground (for horizontal elements).

In many larger arrays, two or more elements must be fed either in phase or out of phase with equal currents, but coupling to other elements may cause their impedances to change unequally—sometimes extremely so. Using the current-forcing method allows the feed system designer to ignore all these effects while guaranteeing equal and correctly phased currents in any combination of 0° and 180° fed elements.

Feeding Elements in Quadrature

Many popular arrays have elements or groups of elements which are fed in quadrature (90° relative phasing). A combination of the forcing method and a simple adjustable network can produce the correct current balance and element phasing.

Suppose that $\frac{1}{4}$-λ lines of the same impedance are connected to two elements. The magnitudes of the element currents equal the voltages at the feed-line inputs, divided by the characteristic impedance of the lines. The currents are both shifted 90° relative to the input voltages. If the two input voltages can be made equal in magnitude but 90° different in phase, the element currents will also be equal and phased at 90°. Many networks will accomplish the desired function, the simplest being the L network. Either a high-pass or low-pass network can be used. A high-pass network will give a phase lead, and a low-pass network causes a phase lag. The low-pass network offers dc continuity, which can be beneficial by eliminating static buildup. Only low-pass networks are described here.

The harmonic reduction properties of low-pass networks should not be a consideration in choosing the network type; antenna system matching components should not be depended upon to achieve an acceptable level of harmonic radiation. The quadrature feed system is shown in **Fig 18**.

For element currents of equal magnitude and 90° relative phase, equations for designing the network are

$$X_{ser} = \frac{{Z_0}^2}{R_2} \qquad \text{(Eq 14)}$$

$$X_{sh} = \frac{{Z_0}^2}{X_2 - R_2} \qquad \text{(Eq 15)}$$

where

X_{ser} = the reactance of the series component
X_{sh} = the reactance of the shunt component
Z_0 = the characteristic impedance of the $\frac{1}{4}$-λ lines
R_2 = the feed-point resistance of element 2
X_2 = the feed-point reactance of element 2

R_2 and X_2 may be calculated from Eqs 21 and 22, presented later. If X_{ser} or X_{sh} is positive, that component is an inductor; if negative, a capacitor. In most practical arrays, X_{ser} is an inductor, and X_{sh} is a capacitor.

Unlike the current-forcing methods, the output-to-input voltage transformation and the phase shift of an L network *do* depend on the feed-point impedances of the array elements. So the impedances of the elements, when coupled to each other and while being excited to have the proper currents, must be known in order to design a proper L network. Methods for determining the impedance of one element in the presence of others are presented in later sections.

Suffice it to say here that the self-impedances of the elements and their mutual impedance must be known in order to calculate the element feed-point impedances. In practice, if simple dipoles or verticals are used, a rough estimation of self- and mutual impedances is generally enough to provide a starting point for determining the component values. Then the components may be adjusted for the desired array performance.

The Magic Bullet

Two elements could be fed in quadrature without the necessity to determine self- and mutual impedances if a quadrature forcing network could be found. This passive network would have any one of the following characteristics, but the condition must be *independent of the network load impedance:*

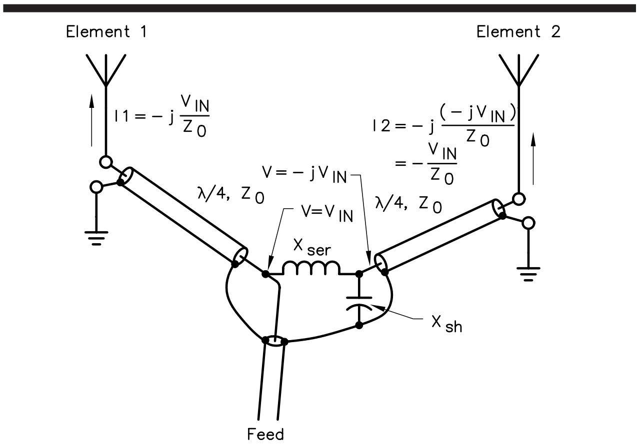


Fig 18—Quadrature feed system. Equations in the text permit calculation of values for the L network components, X_{ser} and X_{sh}.

1) The output voltage is equal in amplitude and 90° delayed or advanced in phase relative to the input voltage.
2) The output current is equal in amplitude and 90° delayed or advanced in phase relative to the input current.
3) The output voltage is in phase or 180° out of phase with the input current, and the magnitude of the output voltage is related to the magnitude of the input current by a constant.
4) The output current is in phase or 180° out of phase with the input voltage, and the magnitude of the output current is related to the magnitude of the input voltage by a constant.

Such a network would be the *magic bullet* to extend the forcing method to quadrature feed systems. Lewallen has looked long and hard for this magic bullet without success. Among the many unsuccessful candidates is the 90° hybrid coupler. Like the Wilkinson divider, the hybrid coupler is a useful port-to-port isolation device that does not accomplish the needed function for this application. The feeding of amateur arrays could be greatly simplified by use of a suitable network. Any reader who is aware of such a network is encouraged to publish it in amateur literature, or to contact Lewallen or the editors of this book.

PATTERN AND GAIN CALCULATION

The following equations are derived from those published by Brown in 1937. Findings from Brown's and later works are presented in concise form by Jasik. Equivalent equations may be found in other texts, such as *Antennas* by Kraus. (See the Bibliography at the end of this chapter.) The equations in this part will enable the mathematically inclined amateur armed with a calculator or computer to determine patterns, actual gains, and front-to-back or front-to-side ratios of two-element arrays. Although only two-element arrays are presented in detail in this part, the principles hold for larger arrays.

The importance of equal element currents (assuming identical elements) in obtaining the best possible nulls was explained earlier, dissimilar current distributions notwithstanding. Maximum forward gain is obtained usually, if not always, for two-element arrays when the currents are equal. Therefore, most of the equations in this part have been simplified to assume that equal element feed-point currents are produced. Just how this can be accomplished for many common array types has already been described briefly, and is covered in more detail later in this chapter. Equations that include the effects of unequal feed-point currents are also presented later in this chapter.

The equations given below are valid for horizontal or vertical arrays. However, ground-reflection effects must be taken into account when dealing with horizontal arrays, doubling the number of elements, which must be dealt with. In fact, the impedance and vertical radiation patterns of horizontal arrays over a reflecting surface (such as the ground) can be derived by treating the images as additional array elements.

For two-element arrays of identical elements with equal element currents, the field strength gain at a distant point relative to a single similar element is

$$\mathrm{FSG} = 10\log\frac{(R_R + R_L)\left[1+\cos(S\cos\theta+\phi_{12})\right]}{(R_R+R_L)+R_m\cos\phi_{12}} \qquad \text{(Eq 16)}$$

where

FSG = field strength gain, dB
R_R = radiation resistance of a single isolated element
R_L = loss resistance of a single element
S = element spacing in degrees
θ = direction from array (see **Fig 19**)
ϕ_{12} = phase angle of current in element 2 relative to element 1. ϕ_{12} is negative if element 2 is delayed (lagging) relative to element 1
R_m = mutual resistance between elements (see **Fig 20**).

The Gain Equation

The gain value from Eq 16 is the *power gain* in dB, which equals the *field strength* gain in dB. Eq 16 should not be confused with equations used to calculate only the *shape* of the pattern. The above equation gives not only the shape of the pattern, but also the actual gain at each angle, relative to a single element.

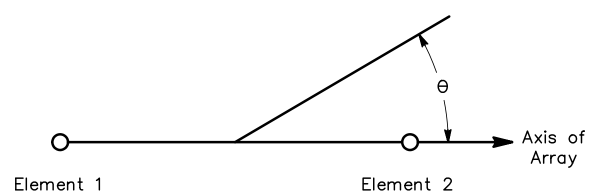


Fig 19—Definition of the angle θ for pattern calculation.

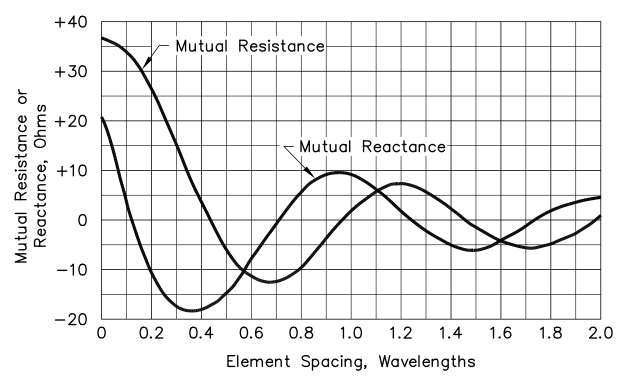


Fig 20—Mutual impedance between two parallel 1/4 λ vertical elements. Multiply the resistance and reactance values by two for 1/2-λ dipoles. Values for vertical elements that are between 0.15 and 0.25 λ high may be approximated by multiplying the given values by R_R/36, where R_R is the radiation resistance of the vertical given by graphs in Chapter 2.

The quantity for which the logarithm is taken in Eq 16 is composed of two major parts,

$$1 + \cos (S \cos \theta + \phi_{12}) \qquad \text{(Term 1)}$$

which relates to field reinforcement or cancellation, and

$$\frac{R_L + R_L}{(R_R + R_L) + R_m \cos \phi_{12}} \qquad \text{(Term 2)}$$

which is the gain change caused by mutual coupling. It is informative to look at each of these terms separately, to see what effect they have on the overall gain.

If there were no mutual coupling at all, Eq 16 would reduce to

$$FSG = 10 \log [1 + \cos (S \cos \theta + \phi_{12})] \qquad \text{(Eq 17)}$$

The term

$$\cos (S \cos \theta + \phi_{12}) \qquad \text{(Term 3)}$$

can assume values from –1 to +1, depending on the element spacing, current phase angle, and direction from the array. In the directions in which the term is –1, the gain becomes zero; a null occurs. Where the term is equal to +1, a maximum gain of

$$FSG = 10 \log 2 = 3 \text{ dB} \qquad \text{(Eq 18)}$$

occurs. This is the same conclusion reached earlier (Eq 13). If the element spacing is insufficient, the term will fail to reach –1 or +1 in any direction, resulting in incomplete nulls or reduced gain, or both. Analysis of the spacing required for the term to reach –1 and +1 results in the graphs of Fig 11.

Analyzing array operation without mutual coupling is not simply an intellectual exercise, even though mutual coupling is present in all arrays. There are some circumstances that will make the mutual coupling portion of the gain equation equal, or very nearly equal, to one. Term 2 above will equal one if

$$R_m \cos \phi_{12} \qquad \text{(Term 4)}$$

is equal to zero. This will happen if $R_m = 0$, which does occur at an element spacing of about 0.43 λ (see Fig 20). Arrays don't usually have elements spaced at 0.43 λ, but a much more common circumstance can cause the effect of mutual coupling on gain to be zero. Term 4 also equals zero if ϕ_{12}, the phase angle between the element currents, is ± 90°. As a result, the gain of any two-element array with 90° phased elements is 3 dB in the favored directions, provided that the spacing is at least 1/4 λ. The 1/4-λ minimum is dictated by the requirement for full field reinforcement. If the elements are closer together, the gain will be less than 3 dB, as indicated in Fig 11.

Loss Resistance and Antenna Gain

A circumstance that reduces the gain effects of mutual coupling is the presence of high losses. If the loss resistance, R_L, becomes very large, the $R_R + R_L$ part of Term 2 above gets much larger than the $R_m \cos \phi_{12}$ part. Then Term 2, the mutual coupling part of the gain equation, becomes approximately

$$\frac{R_R + R_L}{R_R + R_L} = 1$$

Thus, the gain of any very lossy two-element array is 3 dB relative to a single similar element, providing that the spacing is adequate for full field reinforcement. Naturally, higher losses will always lower the gain relative to a single *lossless* element.

This principle can be used to obtain substantial gain if an inefficient antenna system is in use. The technique is to construct one or more additional closely spaced elements (each with its own ground system), and feed the resulting array with all elements in phase. The array won't have appreciable directivity, but it will have significant gain if the original system is very inefficient. As losses increase, the gain approaches 10 log n, where n is the number of elements—3 dB for two elements. This gain, of course, is relative to the original lossy element, so the system gain is unlikely to exceed that of a single lossless element.

Why does a close-spaced second element provide gain? An intuitive way to understand it is to note that two or more closely spaced in-phase elements behave almost like a single element, because of mutual coupling. However, the ground systems are not coupled, so they behave like parallel resistors. The result is a more favorable ratio of radiation to loss resistance. In an efficient system, which has a favorable ratio to begin with, the improvement is not significant, but it can be very significant if the original antenna is inefficient.

The following example illustrates the use of this technique to improve the performance of a 1.8-MHz antenna system. Suppose the original system consists of a single 50-foot high vertical radiator with a 6-inch effective diameter. This antenna will have a radiation resistance, R_R, of 3.12 Ω at 1.9 MHz. A moderate ground system on a city lot will have a loss resistance, R_L, of perhaps 20 Ω. The efficiency of the antenna will be 3.12/(20 + 3.12) = 13.5%, or –8.7 dB relative to a perfectly efficient antenna.

If a second 50-foot antenna with a similar ground system is constructed just 10 feet away from the first, the mutual resistance between elements will be 3.86 Ω. (Calculation of mutual resistance for very short radiators isn't covered in this chapter, but Brown shows that the mutual resistance between short radiators drops approximately in proportion to the self-resistance of each element.) Putting the appropriate values into Eq 16 shows an array gain of 2.34 dB relative to the original single element.

When the effects of mutual coupling are present, the gain in the favored direction can be greater or less than 3 dB, depending on the sign of Term 4. Analysis becomes easier if the element spacing is assumed to be sufficient for full field reinforcement. If this is true, the gain in the favored direction is

$$\text{FSG} = 10 \log \frac{2(R_R + R_L)}{(R_R + R_L) + R_m \cos \phi_{12}}$$
$$= 3\text{ dB} + 10 \log \frac{R_R + R_L}{(R_R + R_L) + R_m \cos \phi_{12}} \quad \text{(Eq 19)}$$

Note that Term 4 above appears in the denominator of Eq 19. If maximum gain is the goal, this term should be made as negative as possible. One of the more obvious ways is to make ϕ_{12}, the phase angle, be 180°, so that $\cos \phi_{12} = -1$, and space the elements closely to make R_m large and positive (see Fig 20). Unfortunately, close spacing does not permit total field reinforcement, so Eq 19 is invalid for this approach. However, the very useful gain of just under 4 dB is still obtainable with this concept if the loss is kept very low. The highest gains for two-element arrays (about 5.6 dB) occur at close spacings with feed angles just under 180°. All close-spaced, moderate to high-gain arrays are very sensitive to loss, so they generally will produce disappointing results when made with ground-mounted vertical elements.

Here are some examples which illustrate the use of Eq 16. Consider an array of two parallel, 1/4-λ high, ground-mounted vertical elements, spaced 1/2 λ apart and fed 180° out of phase. For this array,

$R_R = 36\ \Omega$
S = 180°
$\phi_{12} = 180°$
$R_m = -6\ \Omega$ (from Fig 20)

R_L must be measured or approximated, measurements being preferred for best accuracy. Suitable methods are described later. Alternatively, R_L can be estimated from graphs of ground-system losses. Probably the most extensive set of measurements of vertical antenna ground systems was published by Brown, et al in their classic 1937 paper. Their data have been republished countless times since, in amateur and other literature. Unfortunately, information is sparse for systems of only a few radials because Brown's emphasis is on broadcast installations. Measurements by Sevick nicely fill this void. From his data, we find that the typical feed-point resistance of a 1/4-λ vertical with four 0.2 to 0.4-λ radials is 65 Ω. (See Fig 24.) The loss resistance is 65 – 36 = 29 Ω. This value is used for the example.

Putting the values into Eq 16 results in

$$\text{FSG} = 10 \log \frac{65\left[1 + \cos\ (180°\ \cos\theta + 180°\right]}{65 + (-6\ \cos\ 180°)}$$

Calculating the result for various values of θ reveals the familiar two-lobed pattern with maxima at 0° and 180°, and complete nulls at 90° and 270°. Maximum gain is calculated from Eq 16 by taking θ as 0°.

$$\text{FSG} = 10 \log \frac{65\,(1+1)}{65+6} = 2.63\text{ dB}$$

In this array, the mutual coupling decreases the gain slightly from the nominal 3-dB figure. The reader can confirm that if the element losses were zero ($R_L = 0$), the gain would be 2.34 dB relative to a similar, lossless element. If the elements were extremely lossy, the gain would approach 3 dB relative to a single similar and very lossy element. The efficiency of the original example elements is 36/65 = 55%, and a single isolated element would have a signal strength of 10 log 36/65 = –2.57 dB relative to a lossless element. As determined above, this phased array has a gain of 2.63 dB relative to a single 55% efficient element. Comparing the decibel numbers indicates the array performance in its favored directions is approximately the same as a single lossless element.

Changing the phasing of the array to 0° rotates the pattern 90°, and changes the gain to

$$\text{FSG} = 10 \log \frac{65 \times 2}{65 - 6} = 3.43\text{ dB}$$

A system of very lossy elements would give 3 dB gain as before, and a lossless system would show 3.80 dB (each relative to a single similar element). In this case, the mutual coupling increases the gain above 3 dB, but the losses drop it back toward that figure. This effect can be generalized for larger arrays: Increasing loss in a system of n elements tends to move the gain toward 10 log n relative to a single similar (lossy) element, provided that spacing is adequate for full field reinforcement. If the spacing is closer, losses can reduce gain below this value.

MUTUAL COUPLING AND FEED-POINT IMPEDANCE

The feed-point impedances of the elements of an array are important to the design of some of the feed systems presented here. When elements are placed in an array, their feed-point impedances change from the self-impedance values (impedances when isolated from other elements). The following information shows how to find the feed-point impedances of elements in an array.

The impedance of element 1 in a two-element array is given by Jasik as

$$R_1 = R_s + M_{12}\,(R_m \cos \phi_{12} - X_m \sin \phi_{12}) \quad \text{(Eq 20)}$$

$$X_1 = X_s + M_{12}\,(X_m \cos \phi_{12} + R_m \sin \phi_{12}) \quad \text{(Eq 21)}$$

where

R_1 = the feed-point resistance of element 1
X_1 = the feed-point reactance of element 1
R_S = the self-resistance of a single isolated element = radiation resistance R_R + loss resistance R_L
X_S = the self-reactance of a single isolated element
M_{12} = the magnitude of current in element 2 relative to that in element 1
ϕ_{12} = the phase angle of current in element 2 relative to that in element 1
R_m = the mutual resistance between elements 1 and 2
X_m = the mutual reactance between elements 1 and 2

For element 2,

$$R_2 = R_S + M_{21} (R_m \cos \phi_{21} - X_m \sin \phi_{21}) \quad \text{(Eq 22)}$$

$$X_2 = X_S + M_{21} (X_m \cos \phi_{21} + R_m \sin \phi_{21}) \quad \text{(Eq 23)}$$

where

$$M_{21} = \frac{1}{M_{12}}$$

$$\phi_{21} = -\phi_{12}$$

and other terms are as defined above.

Equations for the impedances of elements in larger arrays are given later.

Two Elements Fed Out of Phase

Consider the earlier example of a two-element array of $^1/_4$-λ verticals spaced $^1/_2$ λ apart and fed 180° out of phase. To find the element feed-point impedances, first the values of R_m and X_m are found from Fig 20. These are –6 and –15 Ω, respectively. Assuming that the element currents can be balanced and that the desired 180° phasing can be obtained, the feed-point impedance of element 1 becomes

$$R_1 = R_S + 1 [-6 \cos 180° - (-15) \sin 180°] = R_S + 6\ \Omega$$

$$X_1 = X_S + 1 [-15 \cos 180° + (-6) \sin 180°] = X_S + 15\ \Omega$$

Suppose that the elements, when not in an array, are resonant ($X_S = 0$) and that they have good ground systems so their feed-point resistances (R_S) are 40 Ω. The feed-point impedance of element 1 changes from 40 + *j* 0 for the element by itself to 40 + 6 + *j* (0 + 15) = 46 + *j* 15 Ω, because of mutual coupling with the second element. Such a change would be quite noticeable.

The second element in this array would be affected by the same amount, as the elements *look* the same to each other—there is no difference between 180° leading and 180° lagging. Mathematically, the difference in the calculation for element 2 involves changing +180° to –180° in the equations, leading to identical results. Elements fed in phase (ϕ_{12} = 0°) also look the same to each other. So for two-element arrays fed in phase (0°) or out of phase (180°), the feed-point impedances of both elements change by the same amount and in the same direction because of mutual coupling. This is not generally true for a pair of elements that are part of a larger array, as a later example shows.

Two Elements with 90° Phasing

Now see what happens with two elements having a different relative phasing. Consider the popular vertical array with two elements spaced $^1/_4$ λ and fed with a 90° relative phase angle to obtain a cardioid pattern. Assuming equal element currents and $^1/_4$-λ elements, Fig 20 shows that R_m = 20 Ω and X_m = –15 Ω. Use Eqs 19 and 20 to calculate the feed-point impedance of the leading element, and Eqs 21 and 22 for the lagging element.

$$R_1 = R_S + 1 [20 \cos(-90°) - (-15) \sin (-90°)] = R_S - 15\ \Omega$$

$$X_1 = X_S + 1 [-15 \cos (-90°) + 20 \sin (-90°)] = X_S - 20\ \Omega$$

And for the lagging element,

$$R_2 = R_S + 1 [20 \cos 90° - (-15) \sin 90°] = R_S + 15\ \Omega$$

$$X_2 = X_S + 1 [(-15) \cos 90° + 20 \sin 90°] = X_S + 20\ \Omega$$

These values represent quite a change in element impedance from mutual coupling. If each element, when isolated, is 50 Ω and resonant (50 + *j* 0 Ω impedance), the impedances of the elements in the array become 35 – *j* 20 and 65 + *j* 20 Ω. These very different impedances can lead to current imbalance and serious phasing errors, if a casually designed or constructed feed system is used.

Close-Spaced Elements

Another example provides a good illustration of several principles. Consider an array of two parallel $^1/_2$-λ dipoles fed 180° out of phase and spaced 0.1 λ apart. To avoid complexity in this example, assume these dipoles are a free-space $^1/_2$-λ long, which is about 1.4% longer than a thin, resonant dipole. At this spacing, from Fig 20, R_m = 67 Ω and X_m = 7 Ω. (Remember to double the values from the graph of Fig 20 for dipole elements.) For each element,

$$R_1 = R_2 = R_S + 1 [67 \cos 180° - 7 \sin 180°] = R_S - 67\ \Omega$$

$$X_1 = X_2 = X_S + 1 [7 \cos 180° + 67 \sin 180°] = X_S - 7\ \Omega$$

The feed-point impedance of an isolated, free-space $^1/_2$-λ dipole is approximately 74 + *j* 44 Ω. Therefore the elements in this array will each have an impedance of about 74 – 67 + *j* (44 – 7) = 7 – *j* 37 Ω! Aside from the obvious problem of matching the array to a feed line, there are some other consequences of such a radical change in the feed-point impedance. Because of the very low feed-point impedance, relatively heavy current will flow in the elements. Normally this would produce a larger field strength, but note from Fig 13 that the element spacing (36°) is far below the 180° required for total field reinforcement. What happens here is that the fields from the elements of this array partially or totally cancel in all directions; there is no direction in which they fully reinforce. As a result, the array produces only moderate gain. Even a few ohms of loss resistance will dissipate a substantial amount of power, reducing the array gain.

This type of array was first described in 1940 by Dr John Kraus, W8JK (see Bibliography). At 0.1-λ spacing, the array will deliver just under 4 dB gain if there is no loss, and just over 3 dB if there is 1-Ω loss per element. The gain drops to about 1.3 dB for 5 Ω of loss per element, and to zero dB at 10 Ω. These figures can be calculated from Eq 16 or read directly from the graphs in Kraus's paper. The modern W8JK array (presented later in this chapter) is based on the array just described, but it overcomes some of the above disadvantages by using four elements instead of two (two pairs of two half waves in phase). Doubling the size of the array provides a theoretical 3 dB gain increase over the above values, and feeding the array as pairs of half waves in phase increases the feed-point impedance to a more reasonable value. However, the modern W8JK array is still sensitive to losses, as described above, because of relatively high currents flowing in the elements.

LARGER ARRAYS

As mentioned earlier, the feed-point impedance of any given element in an array of dipole or ground-mounted vertical elements is altered from its self-impedance by coupling to other elements in the array. Eqs 19 through 22 may be used to calculate the resistive and reactive components of the elements in a two-element array. In a larger array, however, mutual coupling must be taken into account between any given element and all other elements in the array.

Element Feed-Point Impedances

The equations presented in this section may be used to calculate element feed-point impedances in larger arrays. Jasik gives the impedance of an element in an n-element array as follows. For element 1,

$$R_1 = R_{11} + M_{12}(R_{12} \cos \phi_{12} - X_{12} \sin \phi_{12}) + M_{13}(R_{13} \cos \phi_{13} - X_{13} \sin \phi_{13}) + \ldots + M_{1n}(R_{1n} \cos \phi_{1n} - X_{1n} \sin \phi_{1n}) \quad \text{(Eq 24)}$$

$$X_1 = X_{11} + M_{12}(R_{12} \sin \phi_{12} + X_{12} \cos \phi_{12}) + M_{13}(R_{13} \sin \phi_{13} + X_{13} \cos \phi_{13}) + \ldots + M_{1n}(R_{1n} \sin \phi_{1n} + X_{1n} \cos \phi_{1n}) \quad \text{(Eq 25)}$$

For element p,

$$R_p = R_{pp} + M_{p1}(R_{p1} \cos \phi_{p1} - X_{p1} \sin \phi_{p1}) + M_{p2}(R_{p2} \cos \phi_{p2} - X_{p2} \sin \phi_{p2}) + \ldots + M_{pn}(R_{pn} \cos \phi_{pn} - X_{pn} \sin \phi_{pn}) \quad \text{(Eq 26)}$$

$$X_p = X_{pp} + M_{p1}(R_{p1} \sin \phi_{p1} + X_{p1} \cos \phi_{p1}) + M_{p2}(R_{p2} \sin \phi_{p2} + X_{p2} \cos \phi_{p2}) + \ldots + M_{pn}(R_{pn} \sin \phi_{pn} + X_{pn} \cos \phi_{pn}) \quad \text{(Eq 27)}$$

And for element n,

$$R_n = R_{nn} + M_{n1}(R_{n1} \cos \phi_{n1} - X_{n1} \sin \phi_{n1}) + M_{n2}(R_{n2} \cos \phi_{n2} - X_{n2} \sin \phi_{n2}) + \ldots + M_{n\,(n-1)}(R_{n\,(n-1)} \cos \phi_{n\,(n-1)} - X_{n\,(n-1)} \sin \phi_{n\,(n-1)}) \quad \text{(Eq 28)}$$

$$X_n = X_{nn} + M_{n1}(R_{n1} \sin \phi_{n1} + X_{n1} \cos \phi_{n1}) + M_{n2}(R_{n2} \sin \phi_{n2} + X_{n2} \cos \phi_{n2}) + \ldots + M_{n(n-1)}(R_{n(n-1)} \sin \phi_{n\,(n-1)} + X_{n\,(n-1)} \cos \phi_{n\,(n-1)}) \quad \text{(Eq 29)}$$

where

R_{jj} = self resistance of element j
X_{jj} = self reactance of element j
M_{jk} = magnitude of current in element k relative to that in element j
R_{jk} = mutual resistance between elements j and k
X_{jk} = mutual reactance between elements j and k
ϕ_{jk} = phase angle of current in element k relative to that in element j

These are more general forms of Eqs 19 and 20. Examples of using these equations appear in a later section.

Quadrature Fed Elements in Larger Arrays

In some arrays, groups of elements must be fed in quadrature. Such a system is shown in **Fig 21**. The current in each element in the left-hand group equals

$$I1 = -j\frac{V_{in}}{Z_0} \quad \text{(Eq 30)}$$

The current in the elements in the right-hand group equals

$$I2 = -j\frac{V_{out}}{Z_0} \quad \text{(Eq 31)}$$

Thus, if $V_{out} = -jV_{in}$, the right-hand group will have currents equal in magnitude to and 90° delayed from the currents in the left-hand group. The feed-point resistances of the elements have nothing to do with determining the current relationship, except that the relationship between V_{out} and V_{in} is a function of the impedance of the load presented to the L network. That load is determined by the impedances of the elements in the right-hand group.

Values of network components are given by

$$X_{ser} = \frac{Z_0^{\;2}}{\Sigma R_2} \quad \text{(Eq 32)}$$

$$X_{sh} = \frac{Z_0^{\;2}}{\Sigma X_2 - \Sigma R_2} \quad \text{(Eq 33)}$$

where

X_{ser} = the reactance of the series network element
X_{sh} = the reactance of the shunt network element (at the output side)
Z_0 = the characteristic impedance of the element feed lines
ΣR_2 = the sum of the feed-point resistances of all elements connected to the output side of the network

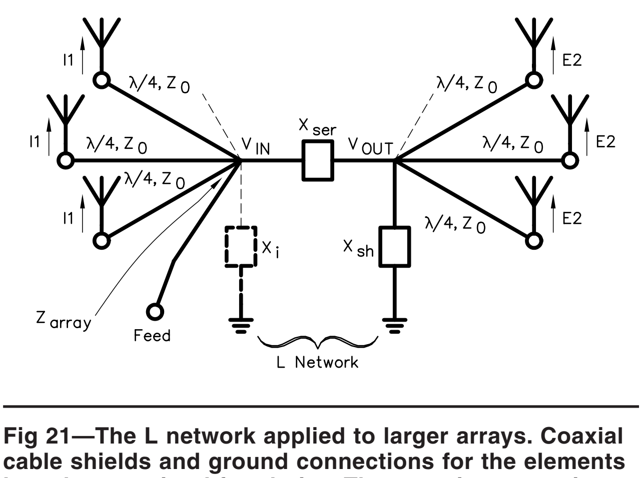


Fig 21—The L network applied to larger arrays. Coaxial cable shields and ground connections for the elements have been omitted for clarity. The text gives equations for determining the component values of X_{ser}, X_{sh} and X_i. X_i is an optional impedance matching component.

ΣX_2 = the sum of the feed-point reactances of all elements connected to the output side of the network

These are more general forms of Eqs 13 and 14. If the value of X_{ser} or X_{sh} is positive, that component is an inductor; if negative, a capacitor.

Array Impedance and Array Matching

Although the impedance matching of an array to the main feed line is not covered in any depth in this chapter, simply adding X_i to the L network, as shown in Fig 21, can improve the match of the array. X_i is a shunt component with reactance, added at the network input. With the proper X_i, the array common-point impedance is made purely resistive, improving the SWR or allowing Q-section matching. X_i is determined from

$$X_i = \frac{Z_0^2}{\Sigma X_1 - \Sigma R_2} \qquad \text{(Eq 34)}$$

where

X_i = the reactance of the shunt network matching element (at the input side)

ΣX_1 = the sum of the feed-point reactances of all elements connected to the input side of the network and other terms are as defined above

If the value of X_i is positive, the component is an inductor; if negative, a capacitor. With the added network element in place, the array common-point impedance is

$$Z_{array} = \frac{Z_0^2}{\Sigma R_1 + \Sigma R_2} \qquad \text{(Eq 35)}$$

where

ΣR_1 = the sum of the feed-point resistances of all elements connected to the input side of the network and other terms are as described above.

X_{ser} and X_{sh} should be adjusted for correct phasing and current balance as described later. They should not be adjusted for the best SWR. X_i, only, is adjusted for the best SWR, and has no effect on phasing or current balance.

CURRENT IMBALANCE AND ARRAY PERFORMANCE

The result of phase error in a driven array was discussed earlier. Changes in phase from the design value produce pattern changes such as shown in Fig 15. Now we turn our attention to the effects of current amplitude imbalance in the elements. This requires the introduction of more general gain equations to take the current ratio into account; the equations given earlier are simplified, based on equal element currents.

Gain, Nulls, and Null Depth

A more general form of Eq 16, taking the current ratios into account, is

$$\text{FSG} = 10\log \frac{(R_R + R_L)\left[1 + M_{12}^2 + 2M_{12}\cos\left(S\cos\theta + \phi_{12}\right)\right]}{(R_R + R_L)(1 + M_{12}^2) + 2M_{12}R_m\cos\phi_{12}} \qquad \text{(Eq 36)}$$

where

FSG = field strength gain relative to a single, similar element, dB

M_{12} = the magnitude of current in element 2 relative to the current in element 1 and other symbols are as defined for Eq 16.

Eq 36 may be used to determine the array field strength at a distant point relative to that from a single similar element for any spacing of two array elements. Now consider arrays where the spacing is sufficient for total field reinforcement or total field cancellation, or both. Fig 13 shows the spacings necessary to achieve these conditions. The curves of Fig 13 show spacings which will allow the term

$\cos(S\cos\theta + \phi_{12})$

to equal its maximum possible value of +1 (total field reinforcement) and minimum possible value of –1 (total field cancellation). In reality, the fields from the two elements cannot add to zero unless this term is –1 *and* the element currents and distributions are equal. For a given set of element currents, the directions in which the term is +1 are those of maximum gain, and the directions in which the term is –1 are those of the deepest nulls.

The elements in many arrays are spaced at least as far apart as given by the two curves in Fig 13. Considerable simplification results in gain calculations for unequal currents if it is assumed that the elements are spaced to satisfy the conditions of Fig 13. Such simplified equations follow.

In the directions of maximum signal,

$$\text{FSG} = 10\log \frac{(R_R + R_L)(1 + M_{12})^2}{(R_R + R_L)(1 + M_{12}^2) + 2M_{12}R_m\cos\phi_{12}} \qquad \text{(Eq 37)}$$

This is a more general form of Eq 19, and is valid provided that the element spacing is sufficient for total field reinforcement. In the directions of minimum gain (nulls),

$$\text{FSG at nulls} = 10\log \frac{(R_R + R_L)(1 - M_{12})^2}{(R_R + R_L)(1 + M_{12}^2) + 2M_{12}R_m\cos\phi_{12}} \qquad \text{(Eq 38)}$$

This equation is valid if the spacing is enough for total field cancellation. The "front-to-null" ratio can be calculated by combining the above two equations.

$$\text{Front-to-null ratio} = 10\log \frac{(1 + M_{12})^2}{(1 - M_{12})^2} \qquad \text{(Eq 39)}$$

This equation is valid if the spacing is sufficient for total field reinforcement *and* cancellation. The equation for forward gain is further simplified for those special cases where

$R_m \cos \phi_{12}$ (Term 5)

is equal to zero. (See the discussion of Eq 16 and Term 4 in the earlier section, "The Gain Equation.")

$$FSG = 10 \log \frac{(1+M_{12})^2}{1+M_{12}^2} \quad \text{(Eq 40)}$$

This equation is valid if the element spacing is sufficient for total field reinforcement.

If an array is more closely spaced than indicated above, the gain will be less, the nulls poorer, or front-to-null ratio worse than given by Eqs 37 through 40. Eq 36 is valid regardless of spacing.

Graphs of Eqs 39 and 40 are shown in **Fig 22**. Note that the "forward gain" curve applies only to arrays for which Term 5, above, equals zero (which includes all two-element arrays phased at 90° and spaced at least $^1/_4\ \lambda$). The curve is useful, however, to get a ballpark idea of the gain of other arrays. The "front-to-null" curve applies to any two-element array, provided that spacing is wide enough for both full reinforcement and cancellation. Fig 22 clearly shows that current imbalance affects the front-to-null ratio much more strongly than it affects forward gain.

If the two elements have different loss resistances (for example, from different ground systems in a vertical array), gain relative to a single *lossless* element can still be calculated

$$FSG = 10 \log \frac{R_R \left[1 + M_{12}^2 + 2\,M_{12} \cos\left(S \cos\theta + \phi_{12}\right)\right]}{(R_R + R_{L1}) + M_{12}^2 (R_R + R_{L2}) + 2\,M_{12}\,R_m \cos\phi_{12}} \quad \text{(Eq 41)}$$

where

the gain is relative to a lossless element

R_{L1} = loss resistance of element 1

R_{L2} = loss resistance of element 2.

Current Errors with Simple Feed Systems

It has already been said that casually designed feed systems can lead to poor current balance and improper phasing. To illustrate just how significant the errors can be, consider various arrays with typical feed systems.

The first array consists of two resonant, $^1/_4$-λ ground-mounted vertical elements, spaced $^1/_4\ \lambda$ apart. Each element has a feed-point resistance of 65 Ω when the other element is open circuited. This is the approximate value when four radials per element are used. In an attempt to obtain 90° relative phasing, element 1 is fed with a line of electrical length L_1, and element 2 is fed with a line 90 electrical degrees longer (L_2). The results appear in **Table 2**.

Not only is the magnitude of the current ratio off by as much as nearly 40%, but the phase angle is incorrect by as much as 30°! The pattern of the array fed with feed system number 1 is shown in **Fig 23**, with a correctly fed array pattern for reference. Note that the example array has only a 9.0 dB front-to-back ratio, although the forward gain is only 0.1 dB more than the correctly fed array. This pattern was calculated from Eq 36. Similar current distributions in the elements are assumed.

Results will be different for arrays with different ground

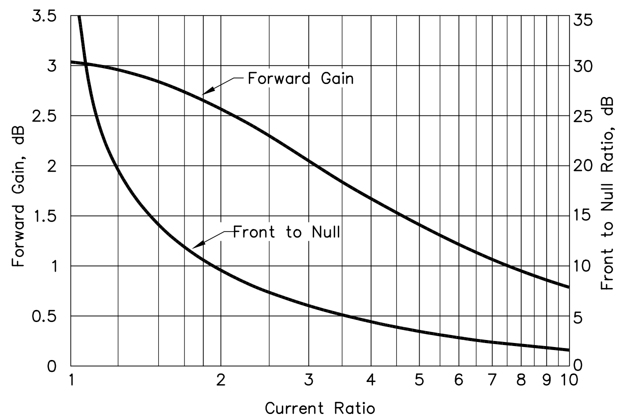


Fig 22—Effect of element current imbalance on forward gain and front-to-null ratio for certain arrays. See text.

Table 2

Two $^1/_4$-λ Vertical Elements with $^1/_4\ \lambda$ Spacing

Feeder system: Line lengths to elements 1 and 2 are given below as L_1 and L_2, respectively. The line length to element 2 is electrically 90° longer than to element 1.

	Feed Lines			***Ele. Feed Point Impedances***		***Ele. Current Ratio***	
No.	*Z_0, Ω*	*L_1, Deg.*	*L_2, Deg.*	*Z_1, Ω*	*Z_2, Ω*	*Mag.*	*Phase, Deg.*
1	50	90	180	50.8 – *j* 6.09	69.8 + *j* 40.0	0.620	–120
2	75	90	180	45.1 –*j* 14.0	73.3 + *j* 24.3	0.973	–108
3	50	180	270	45.7 –*j* 14.1	73.9 + *j* 24.6	0.956	–107
4	75	180	270	51.5 –*j* 11.4	79.4 + *j* 32.4	0.705	–103
5	50	45	135	45.2 –*j* 8.44	68.5 + *j* 28.9	0.859	–120
6	75	45	135	50.2 –*j* 14.9	79.4 + *j* 26.1	0.840	–98
7	Correctly fed			50.0 – *j* 20.0	80.0 + *j* 20.0	1.000	–90

Table 3
Two 1/4-λ Vertical Elements with 1/2 λ Spacing and Different Self-Resistances

Self-resistances: Element 1—50 Ω; Element 2—65 Ω (difference caused by different ground losses). Feeder system: Line lengths to elements 1 and 2 are given below as L_1 and L_2, respectively.

	Feed Lines			*Ele. Feed Point Impedances*		*Ele. Current Ratio*	
No.	*Z_0, Ω*	*L_1, Deg.*	*L_2, Deg.*	*Z_1, Ω*	*Z_2, Ω*	*Mag.*	*Phase, Deg.*
1	Any*	180	180	45.9 – *j* 12.2	56.5 – *j* 18.3	0.800	+3.1
2	50	135	135	43.8 – *j* 11.9	59.7 – *j* 18.6	0.834	–5.8
3	75	135	135	43.2 – *j* 12.5	60.3 – *j* 17.7	0.883	–6.8
4	Any*	270†	270†	44.0 – *j* 15.0	59.0 – *j* 15.0	1.000	0.0
5	50	45	225	53.2 + *j* 12.9	74.8 + *j* 17.1	0.820	–172
6	Any*	180	360	55.6 + *j* 11.0	71.1 + *j* 20.2	0.764	–185
7	Any*	90†	270†	56.0 + *j* 15.0	71.0 + *j* 15.0	1.000	–180

*Both lines must have the same Z_0
†Current forced

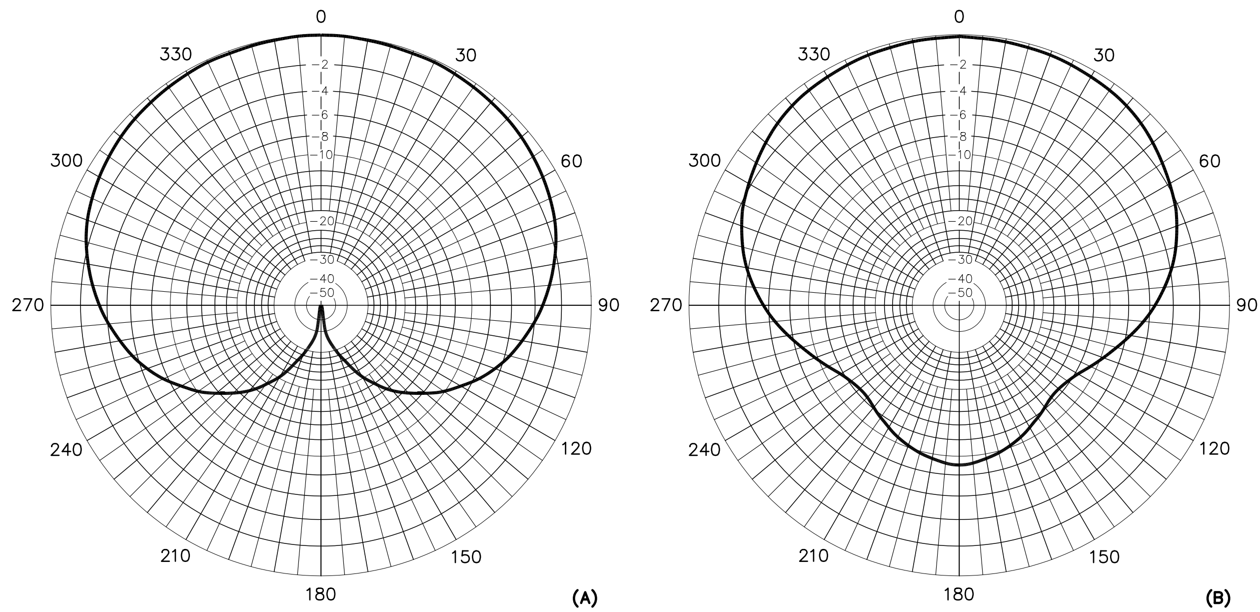


Fig 23—Patterns of an array when correctly fed, A, and when casually fed, B. (See text. Similar current distributions are assumed.) The difference in gain is about 0.1 dB. Gain is referenced to a single similar element; add 3.1 dB to the scale values shown.

systems. For example, if the array fed with feed system 1 had elements with an initial feed-point resistance of 40 Ω instead of 65 Ω, the current ratio would be almost exactly 1—but the phase angle would still be –120°, resulting in poor nulls. The forward gain of the array is +4.0 dB, but the front-to-back ratio is only 11.5 dB.

The advantage of using the current-forcing method to feed arrays of in-phase and 180° out-of-phase elements is shown by the following example. Suppose that the ground systems of two half-wave spaced, 1/4-λ vertical elements are slightly different, so that one element has a feed-point resistance of 50 Ω, the other 65 Ω. (Each is measured when the other element is open circuited.) What happens in this case is shown in **Table 3**.

The patterns of the nonforced arrays are only slightly distorted, with the main deficiency being imperfect nulls. The in-phase array fed with feed system number 1 exhibits a front-to-side ratio of 18.8 dB. The out-of-phase array fed with feed system number 6 has a front-to-side ratio of 17.0 dB. Both these arrays have forward gains very nearly equal to that of a correctly fed array.

Even when the ground systems of the two elements are only slightly different, a substantial current imbalance can occur in in-phase and 180° out-of-phase arrays if casually fed. Two elements with feed-point resistances of 36 Ω and 41 Ω (when isolated), fed with $^1/_2$ and 1 λ of line, respectively, will have a current ratio of 0.881. This is a significant error for a small resistance difference that may be impossible to avoid in practice. As explained earlier, two horizontal elements of different heights, or two elements in many larger arrays, even when fed in phase or 180° out of phase, require more than a casual feed system for correct current balance and phasing.

Phased Array Design Examples

This section, also written by Roy Lewallen, W7EL, presents four examples of practical arrays using the design principles given in previous sections. All arrays are assumed to be made of $^1/_4$-λ vertical elements.

General Array Design Considerations

If the quadrature feed system (Fig 18) is used, the self-impedance of one or more elements must be known. If the elements are common types, such as plain vertical wires or tubes, the impedance can be estimated quite closely from the graphs in Chapter 2. Elements that are close to $^1/_4$-λ high will be near resonance, and calculations can be simplified by adjusting each element to exact resonance (with the other element open-circuited at the feed point) before proceeding. If the elements are substantially less than $^1/_4$ λ high, they will have a large amount of capacitive reactance. This should be reduced in order to keep the SWR on the feed lines to a value low enough to prevent large losses, possible arcing, or other problems. Any tuning or loading done to the elements at the feed point must be in *series* with the elements, so as not to shunt any of the carefully balanced current to ground. A loading coil in series with a short element is permissible, provided that all elements have identical loading coils, but any shunt component at the element feed point must be avoided.

For the following examples, it is assumed that the elements are close to $^1/_4$ λ high and that they have been adjusted for resonance. The radiation resistance of each element is then close to 36 Ω, and the self-reactance is zero because it is resonant.

In any real vertical array, there is ground loss associated with each element. The amount of loss depends on the length and number of ground radials, and on the type and wetness of the ground under and around the antenna. This resistance appears in series with the radiation resistance. The self-resistance is the sum of the radiation resistance and the loss resistance. **Fig 24** gives resistance values for typical ground systems, based on measurements by Sevick (July 1971 and March 1973 *QST*). The values of quadrature feed system components based on Fig 24 will be reasonably close to correct, even if the ground characteristics are somewhat different than Sevick's.

Feed systems for the design example arrays to follow are based on the resistance values given below.

Number of Radials	*Loss Resistance, Ω*
4	29
8	18
16	9
Infinite	0

The mutual impedance of the elements also must be known in order to calculate the impedances of the elements when in the array. The mutual impedance of parallel elements of near-resonant length may be taken from Fig 20. For elements of different lengths, or for unusual shape or orientation, the mutual impedance is best determined by measurement, using measurement methods as given later. Fig 20 suffices for the mutual impedance values in the example arrays.

The matter of matching the array for the best SWR on the feed line to the station is not discussed here. Many of the simpler arrays provide a match that is close to 50 or 75 Ω, so no further matching is required. If better matching is necessary, the appropriate network should be placed in the single feed line running to the station. Attempts to improve the match by adjustment of the phasing L network, antenna lengths, or individual element feeder lengths will ruin the current balance of the array. Information on impedance matching may be found in Chapters 25 and 26.

90° FED, 90° SPACED ARRAY

The feed system for a 90° fed, 90° spaced array is shown in Fig 18. The values of the inductor and capacitor must be calculated, at least approximately. The exact values can be determined by adjustment.

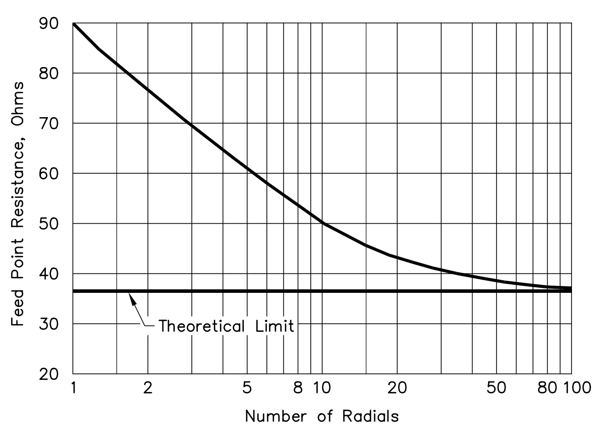


Fig 24—Approximate feed point resistance of a resonant $^1/_4$-λ ground-mounted vertical element versus the number of radials, based on measurements by Jerry Sevick, W2FMI. Moderate length radials (0.2 to 0.4 λ) were used for the measurements. The exact resistance, especially for only a few radials, will depend on the nature of the soil under the antenna.

In this example the elements are assumed to be close to 1/4 λ high, and each is assumed to have been adjusted for resonance with the other element open circuited. If each element has, say, four ground radials, the ground loss resistance is approximately 29 Ω. The self-resistance is 65 Ω. The self-reactance is zero, as the elements are resonant. From Fig 20, the mutual resistance of two parallel 1/4-λ verticals spaced 1/4 λ apart is 20 Ω, and the mutual reactance is –15 Ω. These values are used in Eqs 20 through 23 to calculate the feed-point impedances of the elements. Element currents of equal magnitude are required, so $M_{12} = M_{21} = 1$. The L network causes the current in element 2 to lag that in element 1 by 90°, so $\phi_{12} = -90°$ and $\phi_{21} = 90°$. Summarizing,

$R_S = 65\ \Omega$
$X_S = 0\ \Omega$
$M_{12} = M_{21} = 1$
$\phi_{12} = -90°$
$\phi_{21} = 90°$
$R_m = 20\ \Omega$
$X_m = -15\ \Omega$

Putting these values into Eqs 19 through 22 results in the following values.

$R_1 = 50\ \Omega$
$X_1 = -20\ \Omega$
$R_2 = 80\ \Omega$
$X_2 = 20\ \Omega$

These are the actual impedances at the bases of the two elements when placed in the array and fed properly. It is necessary only to know the impedance of element 2 in order to design the L network, but the impedance of element 1 was calculated here to show how different the impedances are. Next, the impedance of the feed lines is chosen. Suppose the choice is 50 Ω. For Eqs 14 and 15,

$Z_0 = 50\ \Omega$
$R_2 = 80\ \Omega$
$X_2 = 20\ \Omega$

From Eq 14,

$$X_{ser} = \frac{50^2}{80} = 31.3\ \Omega$$

And from Eq 15,

$$X_{sh} = \frac{50^2}{20-80} = -41.7\ \Omega$$

The signs show that X_{ser} is an inductor and X_{sh} is a capacitor. The actual values of L and C can be calculated for the desired frequency by rearranging and modifying the basic equations for reactance.

$$L = \frac{X_L}{2\pi f} \qquad \text{(Eq 42)}$$

$$C = \frac{-10^6}{2\pi f X_c} \qquad \text{(Eq 43)}$$

where

L = inductance, µH
C = capacitance, pF
f = frequency, MHz
X_L and X_C = reactance values, Ω

The negative sign in Eq 43 is included because capacitive reactance values are given here as negative. A similar process is followed to find the values of X_{ser} and X_{sh} for different ground systems and different feed-line impedances. The results of such calculations appear in **Table 4**.

To obtain correct performance, both network components must be adjustable. If an adjustable inductor is not convenient or available, a fixed inductor in series with a variable capacitor will provide the required adjustability. The equivalent reactance should be equal to the value calculated for X_{ser}.

For example, to use the above design at 7.15 MHz, $L_{ser} = 0.697$ µH, and $C_{sh} = 534$ pF. The 0.697 µH inductor (reactance = 31.3 Ω) can be replaced by a 1.39 µH inductor (reactance = approximately 62.6 Ω) in series with a variable capacitor capable of being adjusted on both sides of 711 pF (reactance = –31.3 Ω). The reactance of the series combination can then be varied on both sides of 62.6 – 31.3 = 31.3 Ω. Actually, it might be preferable to use 75-Ω feed line instead of 50 Ω for this array. Table 4 shows that the L network reactances are about twice as great if 75-Ω line is chosen. This means that the required capacitance would be one half as large. Smaller adjustable capacitors are more common, and more compact.

The voltages across the network components are relatively low. Components with breakdown voltages of a few hundred volts will be adequate for a few hundred watts of output power. If fixed capacitors are used, they should be good quality mica or ceramic units.

A THREE-ELEMENT BINOMIAL BROADSIDE ARRAY

An array of three in-line elements spaced 1/2 λ apart and fed in phase gives a pattern that is generally bidirectional. If the element currents are equal, the resulting pattern has a forward gain of 5.7 dB (for lossless elements) but substantial side lobes. If the currents are tapered in a bino-

Table 4
L Network Values for Two Elements 1/4 λ Apart, Fed 90° Out of Phase (Fig 18)

R_S, Ω	*No. of Radials per Element*	*Z_0, Ω*	*X_{ser} Ω*	*X_{sh} Ω*
65	4	50	31.3	–41.7
65	4	75	70.3	–93.8
54	8	50	36.2	–51.0
54	8	75	81.5	–114.8
45	16	50	41.7	–62.5
45	16	75	93.8	–140.6
36	×	50	49.0	–80.6
36	×	75	110.3	–181.5

mial coefficient 1:2:1 ratio (twice the current in the center element as in the two end elements), the gain drops slightly to 5.2 dB, the main lobes widen, and the side lobes disappear.

The array is shown in **Fig 25**. To obtain a 1:2:1 current ratio in the elements, each end element is fed through a $^3/_4$-λ line of impedance Z_0. Line lengths of $^3/_4$-λ are chosen because $^1/_4$-λ lines will not physically reach. The center element is fed from the same point through two parallel $^3/_4$-λ lines of the same characteristic impedance, which is equivalent to feeding it through a line of impedance $Z_0/2$. The currents are thus forced to be in phase and to have the correct ratio.

A FOUR-ELEMENT RECTANGULAR ARRAY

The four-element array shown with its pattern in **Fig 26** has appeared numerous times in amateur publications. However, the accompanying feed systems invariably fail to deliver currents in the proper amounts and phases to the various elements. The array can be correctly fed using the principles discussed in this section.

Elements 1 and 2 can be forced to be in phase and to have equal currents by feeding them through $^3/_4$-λ lines. (Again, $^3/_4$-λ lines are chosen because $^1/_4$-λ lines won't physically reach.) Likewise, the currents in elements 3 and 4 can be forced to be equal and in phase. Elements 3 and 4 are made to have currents of equal amplitude but of 90° phase difference from elements 1 and 2 by use of the quadrature feed system shown in **Fig 27**. The phasing network is the type shown in Fig 18, but Eqs 32 and 33 must be used to calculate the network component values. For this array they are

$$X_{ser} = \frac{{Z_0}^2}{R_3 + R_4} = \frac{{Z_0}^2}{2\,R_3} \qquad \text{(Eq 44)}$$

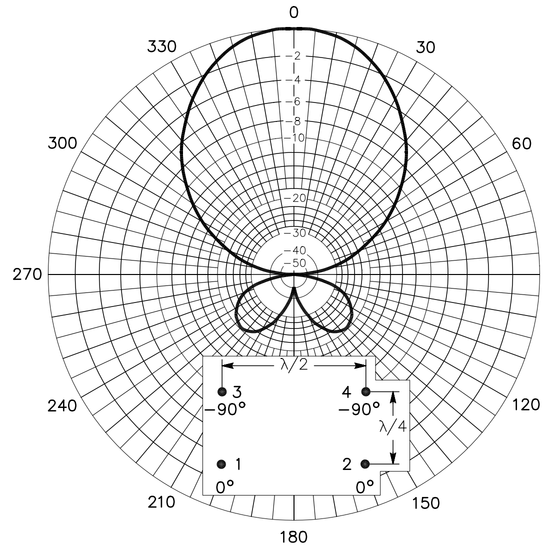


Fig 26—Pattern and layout of the four-element rectangular array. Gain is referenced to a single similar element; add 6.8 dB to the scale values shown.

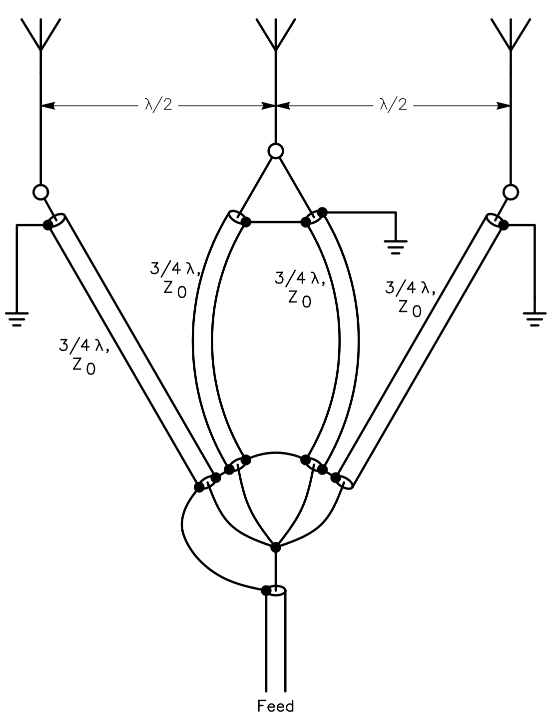


Fig 25—Feed system for the three element 1:2:1 binomial array. All feed lines are ³/₄ electrical wavelength long and have the same characteristic impedance.

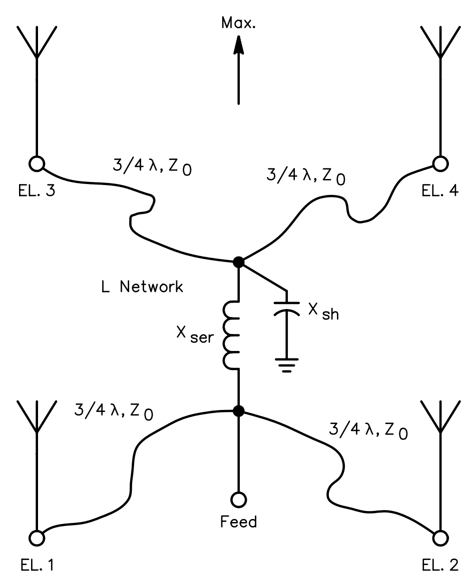


Fig 27—Feed system for the four-element rectangular array. Grounds and cable shields have been omitted for clarity.

$$X_{sh} = \frac{Z_0^2}{X_3 + X_4 - (R_3 + R_4)} = \frac{Z_0^2}{2(X_3 - R_3)} \qquad \text{(Eq 45)}$$

The impedances of elements 3 and 4 will change by the same amount because of mutual coupling. If their ground systems are identical, they will also have equal values of R_L. If the ground systems are different, an adjustment of network values must be made, but the currents in all elements will be equal and correctly phased once the network is adjusted.

Eqs 26 and 27 are used to calculate R_3 and X_3. For element 3, they become

$R_3 = R_S + M_{31}(R_{31} \cos \phi_{31} - X_{31} \sin \phi_{31}) + M_{32}(R_{32} \cos \phi_{32} - X_{32} \sin \phi_{32}) + M_{34}(R_{34} \cos \phi_{34} - X_{34} \sin \phi_{34})$

$X_3 = X_S + M_{31}(R_{31} \sin \phi_{31} + X_{31} \cos \phi_{31}) + M_{32}(R_{32} \sin \phi_{32} + X_{32} \cos \phi_{32}) + M_{34}(R_{34} \sin \phi_{34} + X_{34} \cos \phi_{34})$

where

$M_{31} = M_{32} = M_{34} = 1$
$\phi_{31} = +90°$
$\phi_{32} = +90°$
$\phi_{34} = 0°$
$R_{31} = 20\ \Omega$ (from Fig 20, 0.25-λ spacing)
$X_{31} = -15\ \Omega$ (0.25-λ spacing)
$R_{32} = -10\ \Omega$ (0.56-λ spacing)
$X_{32} = -10\ \Omega$ (0.56-λ spacing)
$R_{34} = -6\ \Omega$ (0.50-λ spacing)
$X_{34} = -15\ \Omega$ (0.50-λ spacing)

resulting in $R_3 = R_S + 19\ \Omega$ and $X_3 = X_S - 5.0\ \Omega$. R_S and X_S are the self-resistance and self-reactance of a single isolated element. In this example, they are assumed to be the same for all elements. Thus, element 4 will have the same impedance as element 3.

It is now possible to make a table of X_{ser} and X_{sh} values for this array for different ground systems and feed-line impedances. The information appears in **Table 5**. Calculation of actual values of L and C are the same as for the earlier example.

Table 5
L Network Values for the Four-Element Rectangular Array (Fig 27)

R_S, Ω	*No. of Radials per Element*	*Z_0, Ω*	*X_{SER}, Ω*	*X_{SH}, Ω*
65	4	50	14.9	–14.0
65	4	75	33.5	–31.6
54	8	50	17.1	–16.0
54	8	75	38.5	–36.1
34	16	50	19.5	–18.1
34	16	75	43.9	–40.8
36	×	50	22.7	–20.8
36	×	75	51.1	–46.9

THE FOUR-SQUARE ARRAY

A versatile array is one having four elements arranged in a square, commonly called the *four-square array*. The array layout and its pattern are shown in **Fig 28**. This array has several attractive properties:

1) 5.5 dB forward gain over a single similar element, for any value of loss resistance;
2) 3 dB or greater forward gain over a 90° angle;
3) 20 dB or better F/B ratio maintained over a 130° angle;
4) symmetry that allows directional switching in 90° increments.

Because of the large differences in element feed-point impedances from mutual coupling, casual feed systems nearly always lead to poor performance of this array. Using the feed system described here, performance is very good, being limited chiefly by environmental factors. Such an array and feed system have been in use at W7EL for several years.

Although the impedances of only two of the four elements need to be calculated to design the feed system, all element impedances will be calculated to show the wide differences in value. This is done by using Eqs 24 through 29, with the following values for the variables.

$M_{jk} = 1$ for all j and k
$R_{12} = R_{21} = R_{13} = R_{31} = R_{24} = R_{42} = R_{34} = R_{43} = 20\ \Omega$ (from Fig 20, 0.25-λ spacing)
$X_{12} = X_{21} = X_{13} = X_{31} = X_{24} = X_{42} = X_{34} = X_{43} = -15\ \Omega$ (0.25-λ spacing)
$R_{14} = R_{41} = R_{23} = R_{32} = 8\ \Omega$ (0.354-λ spacing)
$X_{14} = X_{41} = X_{23} = X_{32} = -18\ \Omega$ (0.354-λ spacing)
$\phi_{12} = \phi_{13} = \phi_{24} = \phi_{34} = -90°$
$\phi_{21} = \phi_{31} = \phi_{42} = \phi_{43} = 90°$

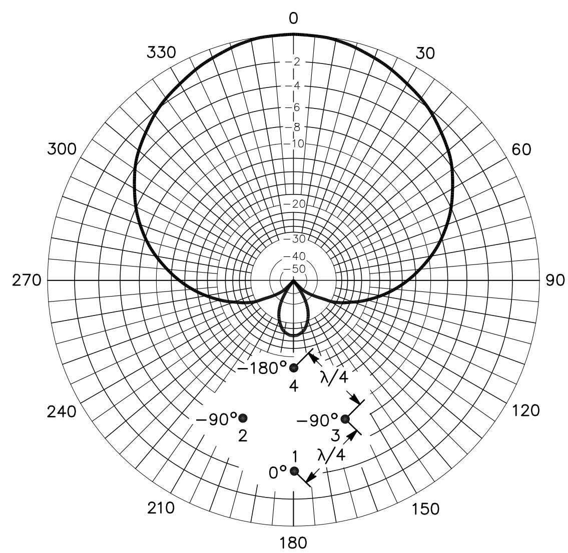


Fig 28—Pattern and layout of the four-square array. Gain is referenced to a single similar element; add 5.5 dB to the scale values shown.

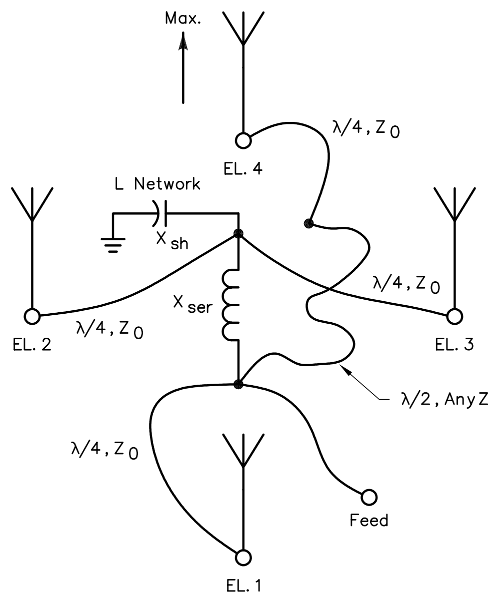


Fig 29—Feed system for the four-square array. Grounds and cable shields have been omitted for clarity.

$\phi_{14} = \phi_{41} = \pm 180°$
$\phi_{23} = \phi_{32} = 0°$

resulting in

$R_1 = R_S - 38\ \Omega$
$X_1 = X_S - 22\ \Omega$
$R_2 = R_3 = R_S + 8\ \Omega$
$X_2 = X_3 = X_S - 18\ \Omega$
$R_4 = R_S + 22\ \Omega$
$X_4 = X_S + 58\ \Omega$

where R_S and X_S are the resistance and reactance of a single element when isolated from the array.

If element 1 had a perfect ground system and were resonant (a self-impedance of $36 + j\,0\ \Omega$), in the array it would have a feed-point impedance of $36 - 38 - j\,22 = -2 - j\,22\ \Omega$. The negative resistance means that it would bedelivering power *into* the feed system. This can, and does, happen in some phased arrays, and is a perfectly legitimate result. The power is, of course, coupled into it from the other elements by mutual coupling. Elements having impedances of precisely zero ohms could have the feed line short circuited at the feed point without effect; that is what a parasitic element is. This is yet another illustration of the error of trying to deliver equal *powers* to the elements.

Table 6
L Network Values for the Four-Square Array (Fig 29)

R_S, Ω	No. of Radials per Element	Z_0, Ω	X_{SER}, Ω	X_{SH}, Ω
65	4	50	17.1	–13.7
65	4	75	38.5	–30.9
54	8	50	20.2	–15.6
54	8	75	45.4	–35.2
45	16	50	23.6	–17.6
45	16	75	53.1	–39.6
36	×	50	28.4	–20.2
36	×	75	63.9	–45.4

The basic system for properly feeding the four-square array is shown in **Fig 29**. Foamed-dielectric cable must be used for the $^1/_4$-λ lines. The velocity factor of solid dielectric cable is lower, making an electrical $^1/_4$ λ of that type physically too short to reach. Elements 2 and 3 are forced to have equal and in-phase currents regardless of differences in ground systems. Likewise, elements 1 and 4 are forced to have equal, 180° out-of-phase currents, in spite of extremely different feed-point impedances. The 90° phasing between element pairs is accomplished, as before, by an L network.

Eqs 44 and 45 may be used directly to generate a table of network element values for this array. For this array the values of resistance and reactance for element 3 are as calculated above.

$R_3 = R_S + 8\ \Omega$
$X_3 = X_S - 18\ \Omega = -18\ \Omega$

(Because each element was resonated when isolated from the other elements, X_s equals 0.) **Table 6** shows values of L-network components for various ground systems and feed-line impedances.

This array is more sensitive to adjustment than the two-element 90° fed, 90° spaced array. Adjustment procedures and a method of remotely switching the direction of this array are described in the section that follows.

Practical Aspects of Phased Array Design

With almost any type of antenna system, there is much that can be learned from experimenting with, testing, and using various array configurations. In this section, Roy Lewallen, W7EL, shares the benefit of years of his experience from actually building, adjusting, and using phased arrays. There is much more work to be done in most of the areas covered here, and Roy encourages the reader to build on this work.

Adjusting Phased Array Feed Systems

If a phased array is constructed only to achieve forward gain, adjusting it is seldom worthwhile. This is because the forward gain of most arrays is quite insensitive to either the magnitude or phase of the relative currents flowing in the elements. If, however, good rejection of unwanted signals is desired, adjustment may be required.

The in-phase and 180° out-of-phase current-forcing methods supply very well-balanced and well-phased currents to the elements without adjustment. If the pattern of an array fed using this method is unsatisfactory, this is generally the result of environmental differences; the elements, furnished with correct currents, do not generate correct fields. Such an array can be optimized in a single direction, but a more general approach than the current-forcing method must be taken. Some possibilities are described by Paul Lee and Forrest Gehrke (see Bibliography).

Unlike the current-forcing methods, the quadrature feed systems described earlier in this chapter are dependent on the element self and mutual impedances. The required L network component values can be computed to a high level of precision, but the results are only as good as the knowledge of the relevant impedances. A practical approach is to estimate the impedances or measure them with moderate accuracy, and adjust the network for the best performance. Simple arrays, such as the two-element 90° fed and spaced array, may be adjusted as follows.

Place a low-power signal source at a distance from the array (preferably several wavelengths), in the direction of the null. While listening to the signal on a receiver connected to the array, alternately adjust the two L-network components for the best rejection of the signal.

This has proved to be a very good way to adjust two-element arrays. However, variable results were obtained when a four-square array was adjusted using this technique. The probable reason is that more than one combination of current balance and phasing will produce a null in a given direction. But the overall array pattern is different for each combination. So a different method must be used for adjusting more complex arrays. This involves actually measuring the element currents one way or another, and adjusting the network until the currents are correct.

MEASURING ELEMENT CURRENTS

The element currents can be measured two ways. One way is to measure them directly at the element feed points, as shown in **Fig 30**. A dual-channel oscilloscope is required to monitor the currents. This method is the most accurate, and it provides a direct indication of the actual relative magnitudes and phases of the element currents. The current probe is shown in **Fig 31**.

Instead of measuring the element currents directly, they may be indirectly monitored by measuring the voltages on the feed lines an electrical $^1/_4$ or $^3/_4$ λ from the array. The voltages at these points are directly proportional to the element currents. All the example arrays presented earlier (Figs 18, 21, 25, 27 and 29) have $^1/_4$ or $^3/_4$ λ lines from all elements to a common location, making this measurement method convenient. The voltages may be observed with a dual-channel oscilloscope, or, to adjust for equal-magnitude currents and 90° phasing, the test circuit shown in **Fig 32** may be used.

The test circuit is connected to the feed lines of two elements which are to be adjusted for 90° phasing (such as elements 1 and 2, or 2 and 4 of the four-square array of Fig 29). Adjust the L-network components alternately until both meters read zero. Proper operation of the test circuit may be verified by disconnecting one of the inputs. The *phase* output should then remain close to zero. If not, there is an undesirable imbalance in the circuit, which must be corrected. Another means of verification is to first adjust the L network so the tester indicates correct phasing (zero volts at the *phase* output). Then reverse the tester input connections to the elements. The *phase* output should remain close to zero.

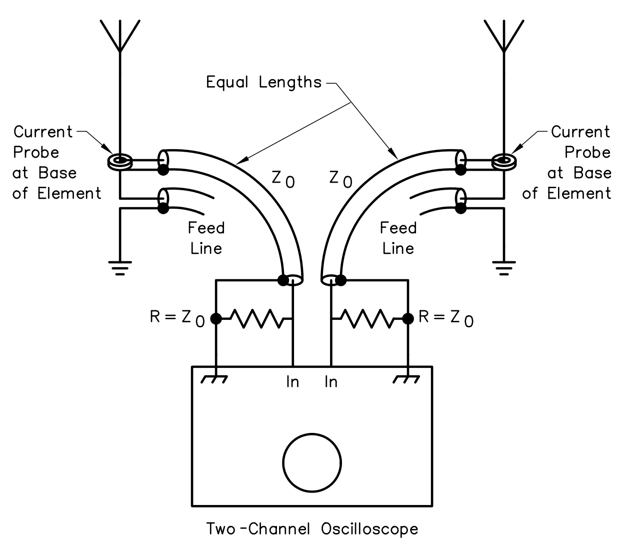


Fig 30—One method of measuring element currents in a phased array. Details of the current probe are given in Fig 31. Caution: Do not run high power to the antenna system for this measurement, or damage to the test equipment may result.

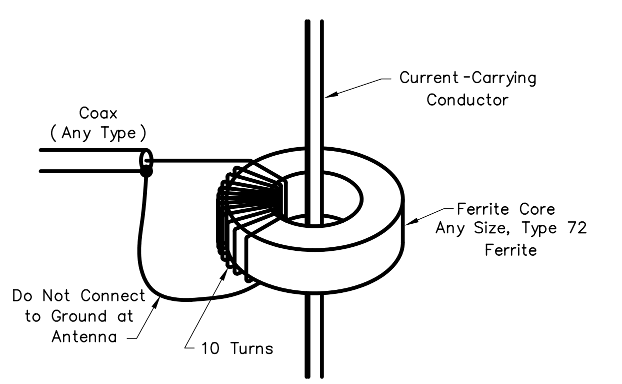


Fig 31—The current probe for use in the test setup of Fig 30. The ferrite core is of type 72 material, and may be any size. The coax line must be terminated at the opposite end with a resistor equal to its characteristic impedance.

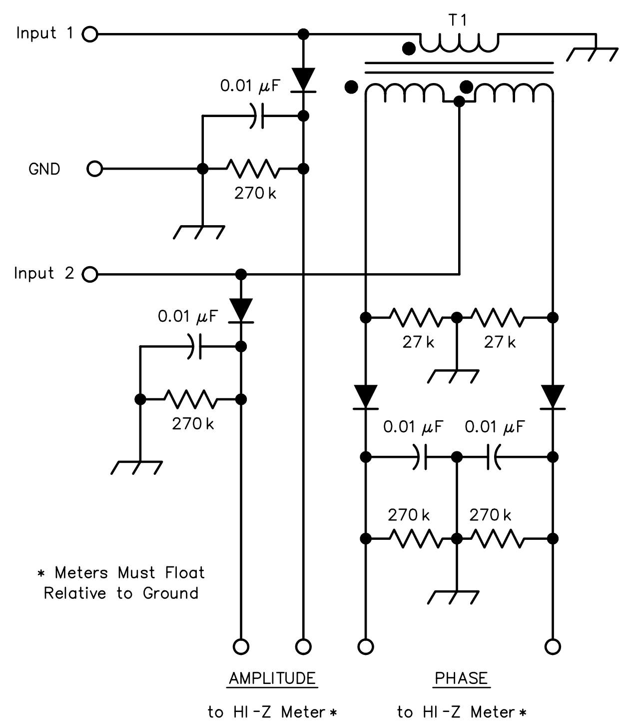


Fig 32—Quadrature test circuit. All diodes are germanium, such as 1N34A, 1N270, or equiv. All resistors are 1/4 or 1/2 W, 5% tolerance. Capacitors are ceramic. Alligator clips are convenient for making the input and ground connections to the array.

T1—7 trifilar turns on an Amidon FT-37-72 or equiv ferrite toroid core.

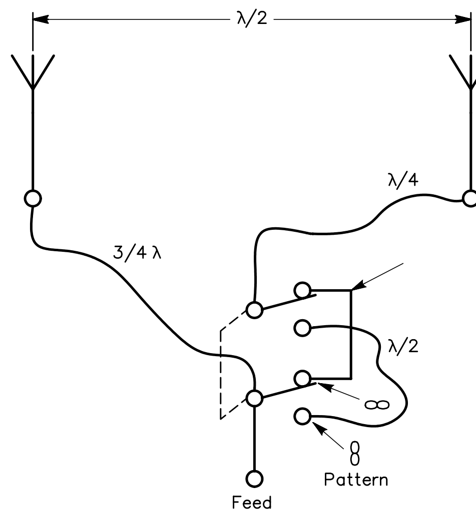


Fig 33—Two-element broadside/end-fire switching. All lines must have the same characteristic impedance. Grounds and cable shields have been omitted for clarity.

DIRECTIONAL SWITCHING OF ARRAYS

One ideal directional-switching method would take the entire feed system, including the lines to the elements, and rotate them. The smallest possible increment of rotation depends on the symmetry of the array—the feed system would need to rotate until the array again *looks* the same to it. For example, any two-element array can be rotated 180° (although that wouldn't accomplish anything if the array was bidirectional to begin with). The four-element rectangular array of Figs 26 and 27 can also be reversed, and the four-square array of Figs 28 and 29 can be switched in 90° increments. Smaller increment switching can be accomplished only by reconfiguring the feed system, including the phase shift network, if used. Switching in smaller increments than dictated by symmetry will create a different pattern in some directions than in others, and must be thoughtfully done to maintain equal and properly phased element currents. The methods illustrated here will deal only with switching in increments related to the array symmetry, except one, a two-element broadside/end-fire array.

In arrays containing quadrature-fed elements, the success of directional switching depends on the elements and ground systems being identical. Few of us can afford the luxury of having an array many wavelengths away from all other conductors, so an array will nearly always perform somewhat differently in each direction. The array, then, should be adjusted when steered in the direction requiring the most signal rejection in the nulls. Forward gain will, for practical purposes, be equal in all the switched directions, since gain is much more tolerant of error than nulls are.

BASIC SWITCHING METHODS

Following is a discussion of basic switching methods, how to power relays through the main feed line, and other practical considerations. In diagrams, grounds are frequently omitted to aid clarity, but connections of the ground conductors must be carefully made. In fact, it is recommended that the ground conductors be switched just as the center conductors are. This is explained in more detail in subsequent text. In all cases, interconnecting lines must be very short.

A pair of elements spaced $^1/_2$ λ apart can readily be switched between broadside and end-fire bidirectional patterns, using the current-forcing properties of $^1/_4$-λ lines. The method is shown in **Fig 33**. The switching device can be a relay powered via a separate cable or by dc sent along the main feed line.

Fig 34 shows directional switching of a 90° fed, 90° spaced array. The rectangular array of Figs 26 and 27 can be switched in a similar manner, as shown in **Fig 35**.

Switching the direction of an array in increments of 90°, when permitted by its symmetry, requires at least two relays. A method of 90° switching of the four-square array is shown in **Fig 36**.

Powering Relays Through Feed Lines

All of the above switching methods can be implemented

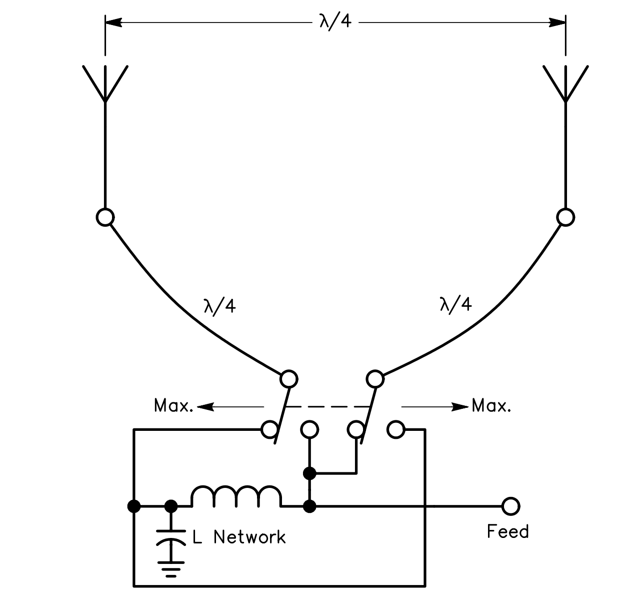


Fig 34—90° fed, 90° phased array reversal switching. All interconnections must be very short. Grounds and cable shields have been omitted for clarity.

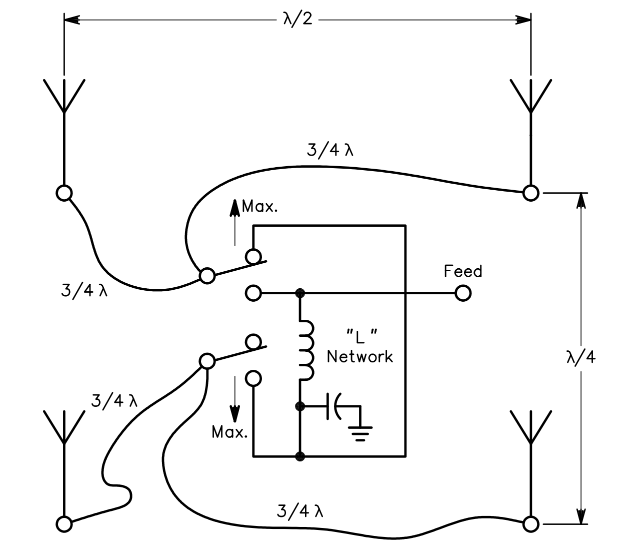


Fig 35—Directional switching of a four-element rectangular array. All interconnections must be very short. Grounds and cable shields have been omitted for clarity.

without additional wires to the switch box. A single-relay system is shown in **Fig 37A**, and a two-relay system in Fig 37B. Small 12 or 24-V dc power relays can be used in either system at power levels up to at least a few hundred watts. Do not attempt to change directions while transmitting, however. Blocking capacitors C1 and C2 should be good quality ceramic or transmitting mica units of 0.01 to 0.1 µF. No problems have been encountered using 0.1 µF, 300-V monolithic ceramic units at RF output levels up to 300 W. C2

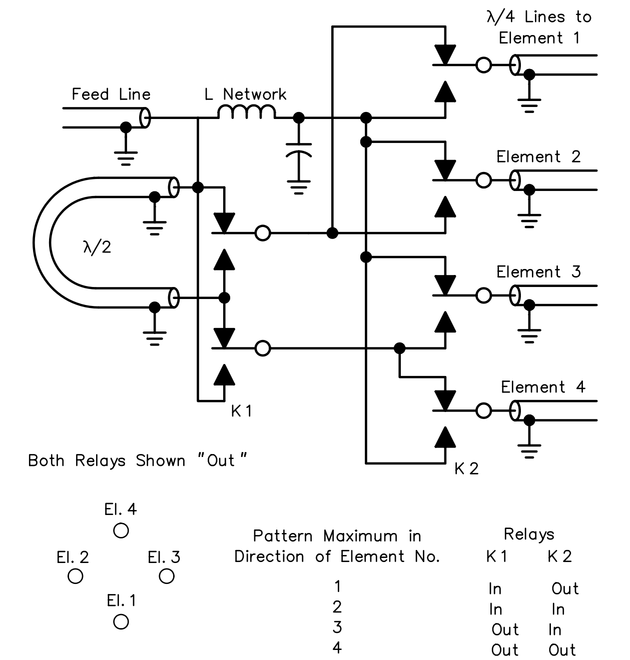


Fig 36—Directional switching of the four-square array. All interconnections must be very short.

may be omitted if the antenna system is an open circuit at dc. C3 and C4 should be ceramic, 0.001 µF or larger.

In Fig 37B, capacitors C5 through C8 should be selected with the ratings of their counterparts in Fig 36A, as given above. Electrolytic capacitors across the relay coils, C9 and C10 in Fig 37B, should be large enough to prevent the relays from buzzing, but not so large as to make relay operation too slow. Final values for most relays will be in the range from 10 to 100 µF. They should have a voltage rating of at least double the relay coil voltage. Some relays do not require this capacitor. All diodes are 1N4001 or similar. A rotary switch may be used in place of the two toggle switches in the two-relay system to switch the relays in the desired sequence.

Although plastic food-storage boxes are inexpensive and durable, using them to contain the direction-switching circuitry might lead to serious phasing errors. If the circuitry is implemented as shown in Figs 33 through 36 and the feed-line grounds are simply connected together, the currents from more than one element share a single conductive path and get phase shifted by the reactance of the wire. As much as 30° of phase shift has been measured at 7 MHz from one side of a plastic box to the other, a distance of only four inches! #12 wire was connecting the two points. Since this experience, twice the number of relay contacts have been used, and the ground conductor of each coaxial cable has been switched right along with the center conductor. A solid metal box might present a path of low enough impedance to prevent the problem. If it does not, the best solution is to use a nonconductive box, and switch the grounds as described.

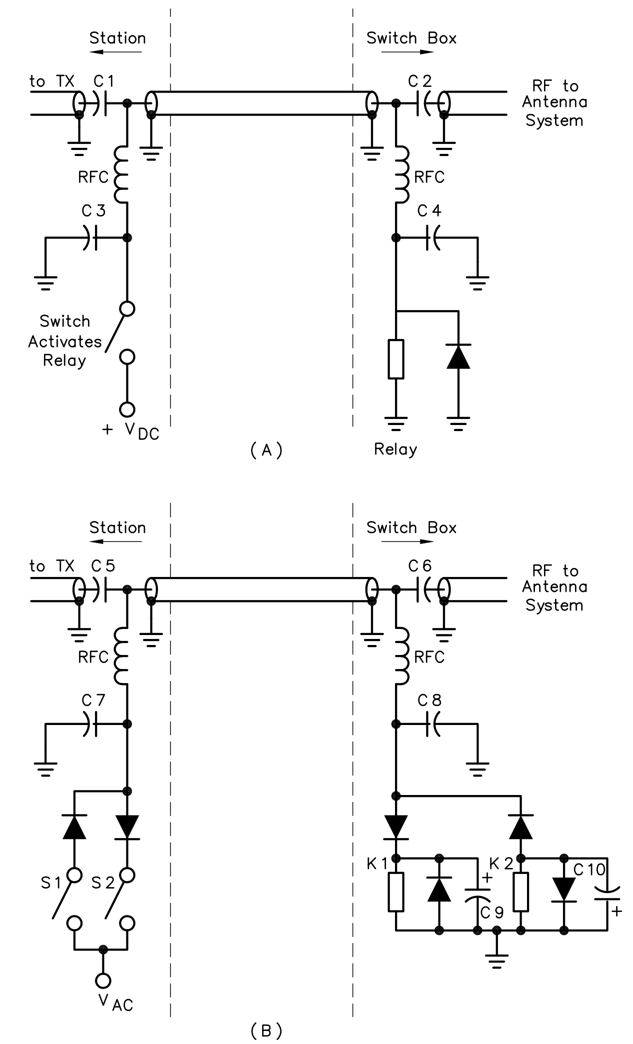


Fig 37—Remote switching of relays. See text for component information. A one-relay system is shown at A, and a two-relay system at B. In B, S1 activates K1, and S2 activates K2.

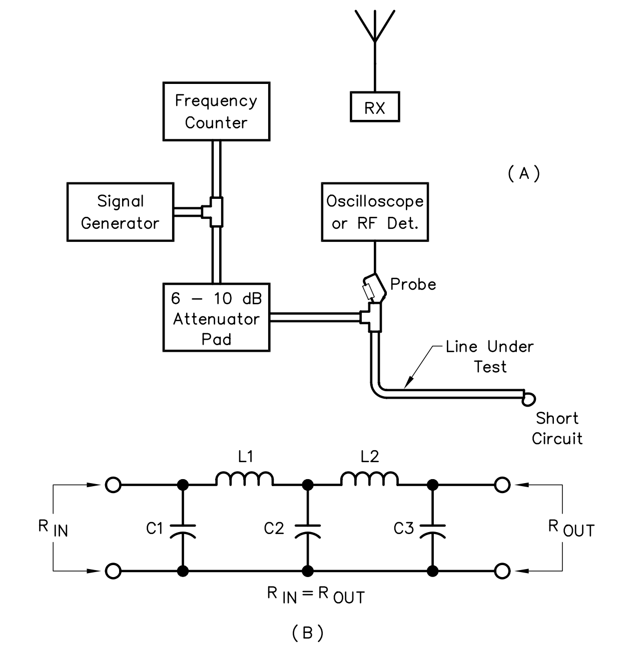


Fig 38—At A, the setup for measurement of the electrical length of a transmission line. The receiver may be used in place of the frequency counter to determine the frequency of the signal generator. The signal generator output must be free of harmonics; the half-wave harmonic filter at B may be used outboard if there is any doubt. It must be constructed for the frequency band of operation. Connect the filter between the signal generator and the attenuator pad.

C1, C3—Value to have a capacitive reactance = R_{IN}.
C2—Value to have a capacitive reactance = $^1/_2$ R_{IN}.
L1, L2—Value to have an inductive reactance = R_{IN}.

MEASURING THE ELECTRICAL LENGTH OF FEED LINES

When using the feed methods described earlier, the feed lines must be very close to the correct length. For best results, they should be correct within 1% or so. This means that a line that is intended to be, say, $^1/_4$ λ at 7 MHz, should actually be $^1/_4$ λ at some frequency within 70 kHz of 7 MHz. A simple but accurate method to determine at what frequency a line is $^1/_4$ or $^1/_2$ λ is shown in **Fig 38A**. The far end of the line is short circuited with a very short connection. A signal is applied to the input, and the frequency is swept until the impedance at the input is a minimum. This is the frequency at which the line is $^1/_2$ λ. Either the frequency counter or the receiver may be used to determine this frequency. The line is, of course, $^1/_4$ λ at one half the measured frequency.

The detector can be a simple diode detector, or an oscilloscope may be used if available. A 6 to 10 dB attenuator pad is included to prevent the signal generator from looking into a short circuit at the measurement frequency. The signal generator output must be free of harmonics. If there is any doubt, an outboard low-pass filter, such as a half-wave harmonic filter, should be used. The half-wave filter circuit is shown in Fig 38B, and must be constructed for the frequency band of operation.

Another satisfactory method is to use a noise or resistance bridge at the input of the line, again looking for a low impedance at the input while the output is short circuited. Simple resistance bridges are described in Chapter 27.

Dip oscillators have been found to be unsatisfactory. The required coupling loop has too great an effect on measurements.

MEASURING ELEMENT SELF-IMPEDANCE

The self-impedance of an unbalanced element, such as a vertical monopole, can be measured directly at the feed point using an impedance bridge. Commercial noise bridges are available, and noise and RLC bridges for home construction are described in Chapter 27. In the 1990s, portable impedance/SWR-measuring instruments with built-in signal generators and digital readouts have become very popular.

When the measurement is being made, all other elements must be *open circuited*. If the feed point is not readily accessible, the impedance can be measured remotely through one or more half wavelengths of transmission line. Other line lengths may also be used, but then an impedance conversion becomes necessary, such as with a Smith Chart (see Chapter 28) or by using a computer program, such as the *TLW* program on the CD-ROM included with this book.

A balanced antenna, for example a dipole, must be measured through a transmission line to permit insertion of the proper type of balun (see below) unless the impedance meter can be effectively isolated from the ground and nearby objects, including the person doing the measurement. When measuring impedance through a transmission line, the following precautions must be taken to avoid substantial errors.

1) The characteristic impedance of the transmission line should be as close as possible to the impedance being measured. The closer the impedances, the less the sensitivity to feed-line loss and length.
2) Do not use any more $^1/_2$-λ sections of line than necessary. Errors are multiplied by the number of sections. Measurements made through lines longer than 1 λ should be suspect.
3) Use low-loss line. Lossy line will skew the measured value toward the characteristic impedance of the line. If the line impedance is close to the impedance being measured, the effect is usually negligible.
4) If a $^1/_2$-λ section of line or multiple is being used, measure the line length using one of the methods described earlier. Do not try to make measurements at frequencies very far away from the frequency at which the line is the correct length. The sensitivity to electrical line length is less if the line impedance is close to the impedance being measured.
5) If the impedance of a balanced antenna such as a dipole is being measured, the correct type of balun must be used. (See Lewallen on baluns, listed in the Bibliography.) One way to make the proper type of balun is to use coaxial feed line, and pass the line through a large, high permeability ferrite core several times, near the antenna. Or a portion of the line may be wound into a flat coil of several turns, a foot or two in diameter, near the antenna. A third method is to string a large number of ferrite cores over the feed line, as described in Chapter 26. The effectiveness of the balun can be tested by watching the impedance measurement while moving the coax about, and grasping it and letting go. The measurement should not change when this is done.

MEASURING MUTUAL IMPEDANCE

Various methods for determining the mutual impedance between elements have been devised. Each method has advantages and disadvantages. The basic difficulty in achieving accuracy is that the measurement of a small change in a large value is required. Two methods are described here. Both require the use of a calibrated impedance bridge. The necessary calculations require a knowledge of complex arithmetic. If measurements are made through feed lines, instead of directly at the feed points, the precautions listed above must be observed.

Method 1

1) Measure the self-impedance of one element with the second element open circuited at the feed point, or with the second element connected to an open-circuited feed line that is an integral number of $^1/_2$ λ long. This impedance is designated Z_{11}.
2) Measure the self-impedance of the second element with the first element open circuited. This impedance is called Z_{22}.
3) Short circuit the feed point of the second element, directly or at the end of an integral number of $^1/_2$ λ of feed line. Measure the impedance of the first element. This is called Z_{1S}.
4) Calculate the mutual impedance Z_{12}.

$$Z_{12} = \pm\sqrt{Z_{22}(Z_{11} - Z_{1S})} \qquad \text{(Eq 46)}$$

where all values are complex.

Because the square root is extracted, there are two answers to this equation. One of these answers is correct and one is incorrect. There is no way to be sure which answer is correct except by noticing which one is closest to a theoretical value, or by making another measurement with a different method. This ambiguity is one disadvantage of using method 1. The other disadvantage is that the difference between the two measured values is small unless the elements are very closely spaced. This can cause relatively large errors in the calculated value of Z_{12} if small errors are made in the measured impedances. Useful results can be obtained with this method if care is taken, however. The chief advantage of method 1 is its simplicity.

Method 2

1) As in method 1, begin by measuring the self-impedance of one element, with the second element open circuited at the feed point, or with the second element connected to a $^1/_2$-λ (or multiple) open-circuited line. Designate this impedance Z_{11}.
2) Measure the self-impedance of the second element with the first element open circuited. Call this impedance Z_{22}.
3) Connect the two elements together with $^1/_2$ λ of transmission line, and measure the impedance at the feed point of one element. A $^1/_2$-λ line may be added to both elements for this measurement if necessary. That is, the line to element 1 would be $^1/_2$ λ, and the line to element 2 a full wavelength. Be sure to read and observe the precautions necessary when measuring impedance through a transmission line, enumerated earlier. This measured impedance is called Z_{1X}.
4) Calculate the mutual impedance Z_{12}.

$$Z_{12} = Z_{21} = -Z_{1X} \pm \sqrt{(Z_{1X}-Z_{11})(Z_{1X}-Z_{22})} \qquad \text{(Eq 47)}$$

where all values are complex.

Again, there are two answers. But the correct one is generally easier to identify than when method 1 is used. For most systems, Z_{11} and Z_{22} are about the same. If they are, the wrong answer will be about equal to $-Z_{11}$ (or $-Z_{22}$). The correct answer will be about equal to $Z_{11} - 2Z_{1X}$ (or $Z_{22} - 2Z_{1X}$). The advantages of this method are that the correct answer is easier to identify, and that there is a larger difference between the two measured impedances. The disadvantage is that the ½-λ line adds another possible source of error.

The wrong answers from methods 1 and 2 will be different, but the correct answers should be the same. Measure with both methods, if possible. Accuracy in these measurements will enable the builder to determine more precisely the proper values of components for a phasing L network. And with precision in these measurements, the performance features of the array, such as gain and null depth, can be determined more accurately with methods given earlier in this chapter.

Broadside Arrays

Broadside arrays can be made up of collinear or parallel elements or combinations of the two. This section was contributed by Rudy Severns, N6LF.

COLLINEAR ARRAYS

Collinear arrays are always operated with the elements in phase. (If alternate elements in such an array are out of phase, the system simply becomes a harmonic type of antenna.) A collinear array is a broadside radiator, the direction of maximum radiation being at right angles to the line of the antenna.

POWER GAIN

Because of the nature of the mutual impedance between collinear elements, the feed-point resistance (compared to a single element, which is =73 Ω) is increased as shown earlier in this chapter (Fig 9). For this reason the power gain does not increase in direct proportion to the number of elements. The gain with two elements, as the spacing between them is varied, is shown by **Fig 39**. Although the gain is greatest when the end-to-end spacing is in the region of 0.4 to 0.6 λ, the use of spacings of this order is inconvenient constructionally and introduces problems in feeding the two elements. As a result, collinear elements are almost always operated with their ends quite close together—in wire antennas, usually with just a strain insulator between.

With very small spacing between the ends of adjacent elements the theoretical power gain of collinear arrays, assuming the use of #12 copper wire, is approximately as follows:

2 collinear elements—1.6 dB
3 collinear elements—3.1 dB
4 collinear elements—3.9 dB
More than four elements are rarely used.

DIRECTIVITY

The directivity of a collinear array, in a plane containing the axis of the array, increases with its length. Small secondary lobes appear in the pattern when more than two elements are used, but the amplitudes of these lobes are low enough so that they are usually not important. In a plane at right angles to the array the directive diagram is a circle, no matter what the number of elements. Collinear operation, therefore, affects only E-plane directivity, the plane containing the antenna.

When a collinear array is mounted with the elements vertical, the antenna radiates equally well in all geographical directions. An array of such *stacked* collinear elements tends to confine the radiation to low vertical angles.

If a collinear array is mounted horizontally, the directive pattern in the vertical plane at right angles to the array is the same as the vertical pattern of a simple λ/2 antenna at the same height (Chapter 3).

TWO-ELEMENT ARRAYS

The simplest and most popular collinear array is one using two elements, as shown in **Fig 40**. This system is commonly known as *two half-waves in phase*. The directive pattern in a plane containing the wire axis is shown in **Fig 41**. Fig 41 gives superimposed patterns for a dipole and 2, 3 and 4 element collinear arrays. Depending on the conductor size, height, and similar factors, the impedance at the feed point can be expected to be in the range of 4 to 6 kΩ, for wire antennas. If the elements are made of tubing having a low λ/dia (wavelength to diameter) ratio, values as low as 1 kΩ are representative. The system can be fed through an

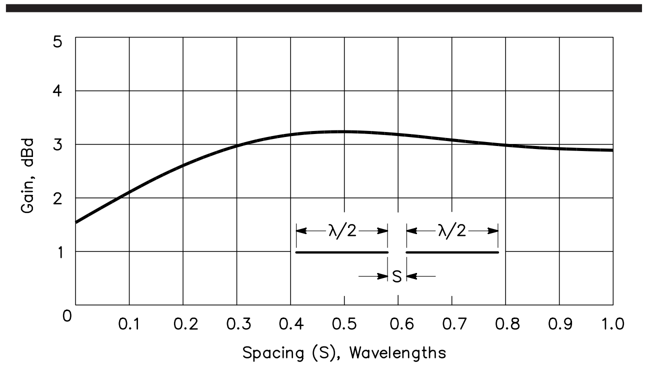


Fig 39—Gain of two collinear ½-λ elements as a function of spacing between the adjacent ends.

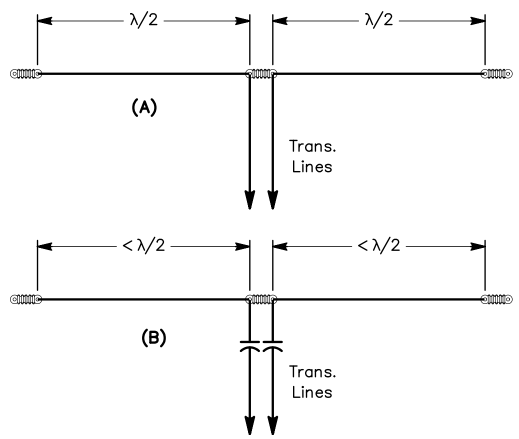


Fig 40—At A, two-element collinear array (two half-waves in phase). The transmission line shown would operate as a tuned line. A matching section can be substituted and a nonresonant line used if desired, as shown at B, where the matching section is two series capacitors.

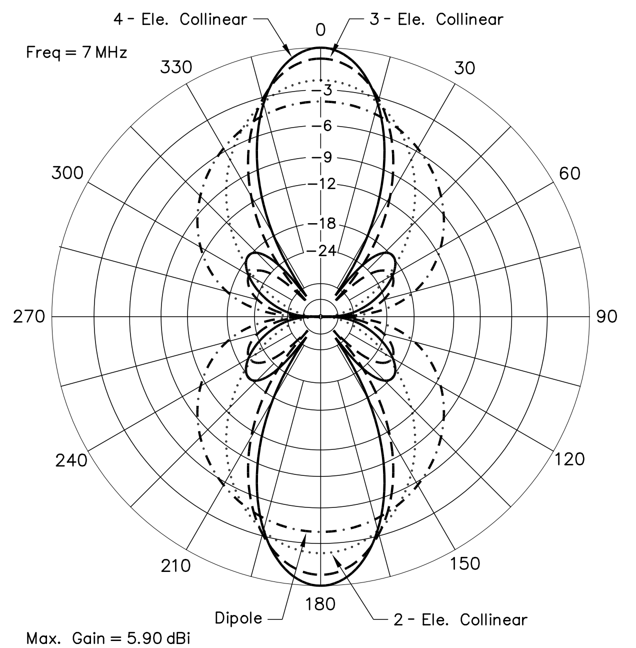


Fig 41—Free-space E-plane directive diagram for dipole, 2, 3 and 4-element collinear arrays. The solid line is a 4-element collinear; the dashed line is for a 3-element collinear; the dotted line is for a 2-element collinear and the dashed-dotted line is for a λ/2 dipole.

open-wire tuned line with negligible loss for ordinary line lengths, or a matching section may be used if desired.

A number of arrangements for matching the feed line to this antenna are described in Chapter 26. If elements somewhat shorter than λ/2 are used, then additional matching schemes can be employed at the expense of a slight reduction in gain. When the elements are shortened two things happen—the impedance at the feed-point drops and the impedance has inductive reactance that can be tuned out with simple series capacitors, as shown in Fig 40B.

Note that these capacitors must be suitable for the power level. Small *doorknob* capacitors such as those frequently used in power amplifiers, are suitable. By way of an example, if each side of a 40-meter 2-element array is shortened from 67 to 58 feet, the feed-point impedance drops from nearly 6000 Ω to about 1012 Ω with an inductive reactance of 1800 Ω. The reactance can be tuned out by inserting 25 pF capacitors at the feed-point. The 1012 Ω resistance can be transformed to 200 Ω using a λ/4 matching section made of 450-Ω ladder line and then transformed to 50 Ω with a 4:1 balun. Shortening the array as suggested reduces the gain by about 0.5 dB.

Another scheme that preserves the gain is to use a 450-Ω λ/4 matching section and shorten the antenna only slightly to have a resistance of 4 kΩ. The impedance at the input of the matching section is then near 50 Ω and a simple 1:1 balun can be used. Many other schemes are possible. The free-space E-plane response for a 2-element collinear array is shown in Fig 41, compared with the responses for more elaborate collinear arrays described below.

THREE- AND FOUR-ELEMENT ARRAYS

In a long wire the direction of current flow reverses in each λ/2 section. Consequently, collinear elements cannot simply be connected end to end; there must be some means for making the current flow in the same direction in all elements. When more than two collinear elements are used it is necessary to connect *phasing* stubs between adjacent elements in order to bring the currents in all elements in phase. In **Fig 42A** the direction of current flow is correct in the two left-hand elements because the shorted λ/4 transmission line (*stub*) is connected between them. This stub may be looked upon simply as the alternate λ/2 section of a long-wire antenna folded back on itself to cancel its radiation. In Fig 42A the part to the right of the transmission line has a total length of three half wavelengths, the center half wave being folded back to form a λ/4 phase-reversing stub. No data are available on the impedance at the feed point in this arrangement, but various considerations indicate that it should be over 1 kΩ.

An alternative method of feeding three collinear elements is shown in Fig 42B. In this case power is applied at the center of the middle element and phase-reversing stubs are used between this element and both of the outer elements. The impedance at the feed point in this case is somewhat over 300 Ω and provides a close match to 300 Ω line. The SWR will be less than 2:1 when 600-Ω line is used. Center feed of this type is somewhat preferable to the arrangement in Fig 42A because the system as a whole is balanced. This assures more uniform power distribution among the elements. In Fig 42A, the right-hand element is likely to receive somewhat less power

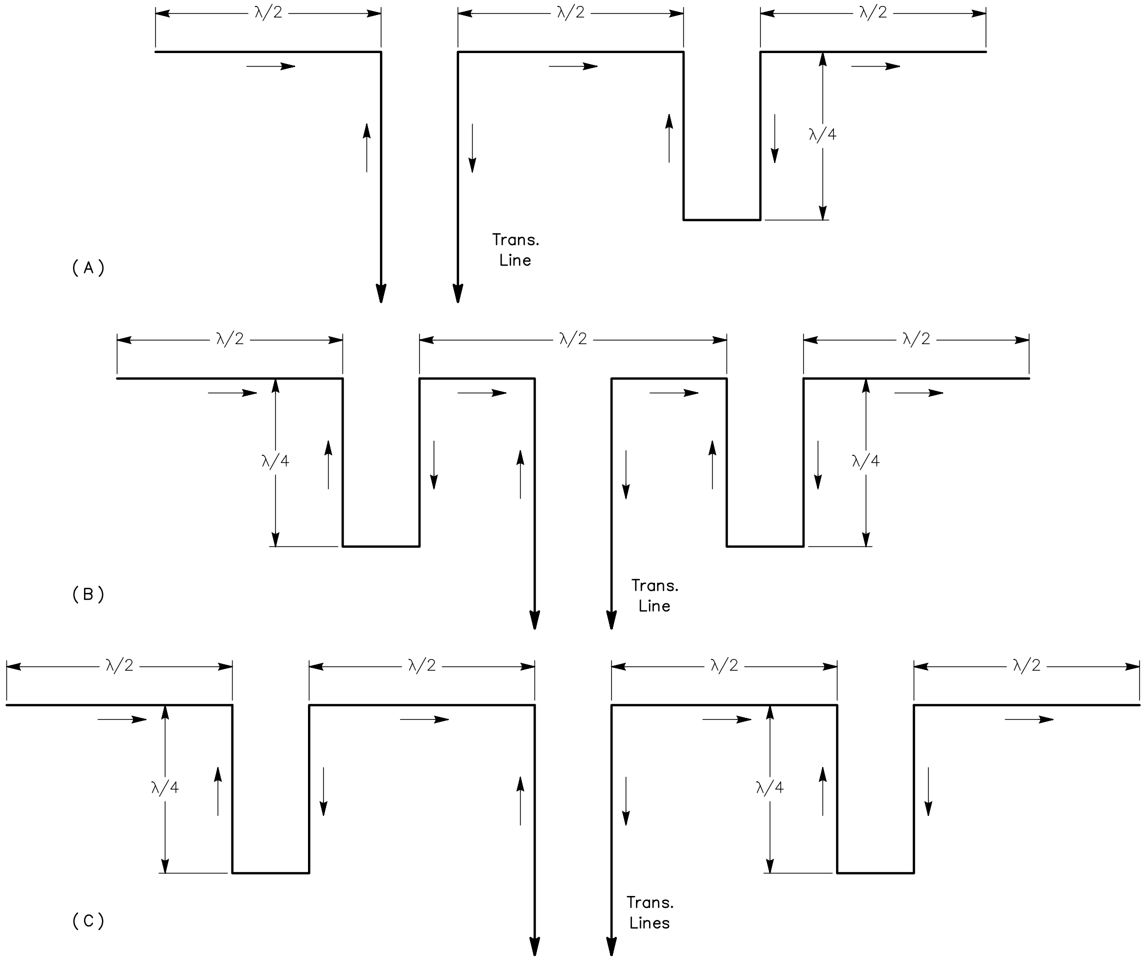


Fig 42—Layouts for 3- and 4-element collinear arrays. Alternative methods of feeding a 3-element array are shown at A and B. These drawings also show the current distribution on the antenna elements and phasing stubs. A matched transmission line can be substituted for the tuned line by using a suitable matching section.

than the other two because a portion of the input power is radiated by the middle element before it can reach the element located at the extreme right.

A four-element array is shown in Fig 42C. The system is symmetrical when fed between the two center elements as shown. As in the three-element case, no data are available on the impedance at the feed point. However, the SWR with a 600 Ω line should not be much over 2:1.

Fig 41 compares the directive patterns of 2, 3 and 4-element arrays. Collinear arrays can be extended to more than four elements. However, the simple two-element collinear array is the type most frequently used, as it lends itself well to multi-band operation. More than two collinear elements are seldom used because more gain can be obtained from other types of arrays.

ADJUSTMENT

In any of the collinear systems described, the lengths of the radiating elements in feet can be found from the formula $468/f_{MHz}$. The lengths of the phasing stubs can be found from the equations given in Chapter 26 for the type of line used. If the stub is open-wire line (500 to 600 Ω impedance) you may assume a velocity factor of 0.975 in the formula for a λ/4 line. On-site adjustment is, in general, an unnecessary refinement. If desired, however, the following procedure may be used when the system has more than two elements.

Disconnect all stubs and all elements except those directly connected to the transmission line (in the case of feed such as is shown in Fig 42B leave only the center element connected to the line). Adjust the elements to resonance, using the still-connected element. When the proper length is determined, cut all other elements to the same length. Make the phasing stubs slightly long and use a shorting bar to adjust their length. Connect the elements to the stubs and adjust the stubs to resonance, as indicated by maximum current in the shorting bars or by the SWR on the transmission line. If more than three or four elements are used it is best to add elements two at a time (one at each end of the array), resonating the system each time before a new pair is added.

THE EXTENDED DOUBLE ZEPP

One method to obtain higher gain that goes with wider spacing in a simple system of two collinear elements is to make the elements somewhat longer than λ/2. As shown in **Fig 43**, this increases the spacing between the two in-phase λ/2 sections at the ends of the wires. The section in the center carries a current of opposite phase, but if this section is short the current will be small; it represents only the outer ends of a λ/2 antenna section. Because of the small current and short length, the radiation from the center is small. The optimum length for each element is 0.64 λ. At greater lengths the system tends to act as a long-wire antenna, and the gain decreases.

This system is known as the *extended double Zepp*. The gain over a λ/2 dipole is approximately 3 dB, as compared with about 1.6 dB for two collinear λ/2 dipoles. The directional pattern in the plane containing the axis of the antenna is shown in **Fig 44**. As in the case of all other collinear arrays, the free-space pattern in the plane at right angles to the antenna elements is the same as that of a λ/2 antenna—circular.

This antenna is not resonant at the operating frequency so that the feed-point impedance is complex (R ± *j* X). A typical example of the variation of the feed-point imped-

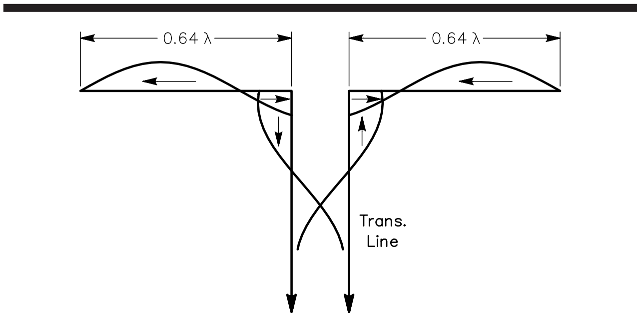


Fig 43—The extended double Zepp. This system gives somewhat more gain than two λ-sized collinear elements.

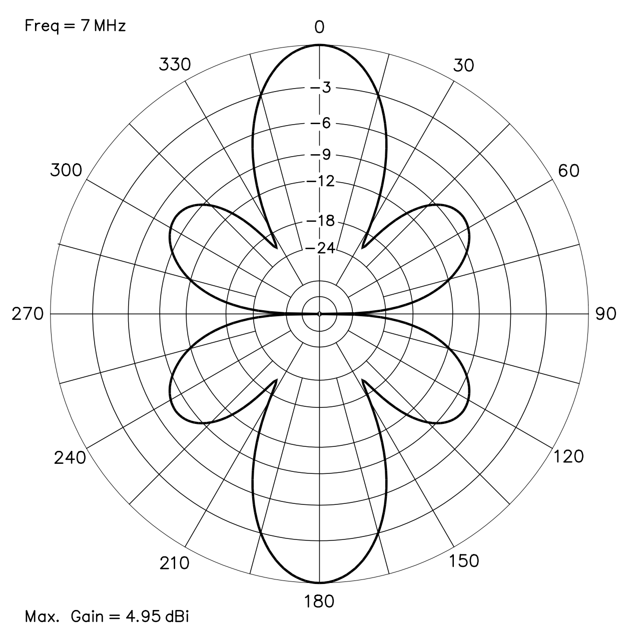


Fig 44—E-plane pattern for the extended double Zepp of Fig 43. This is also the horizontal directional pattern when the elements are horizontal. The axis of the elements lies along the 90°-270° line. The free-space array gain is approximately 4.95 dBi.

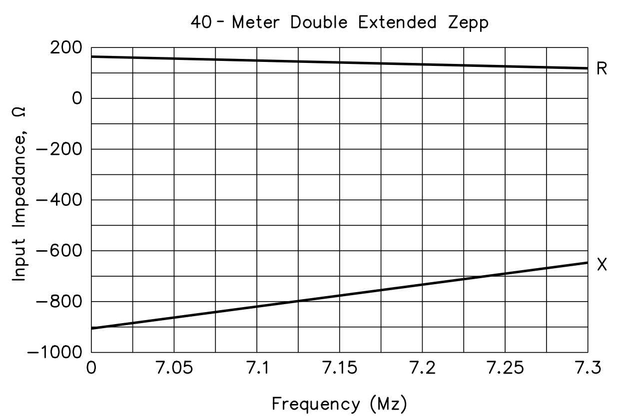


Fig 45—Resistive and reactive feed-point impedance of a 40-meter extended double Zepp in free space.

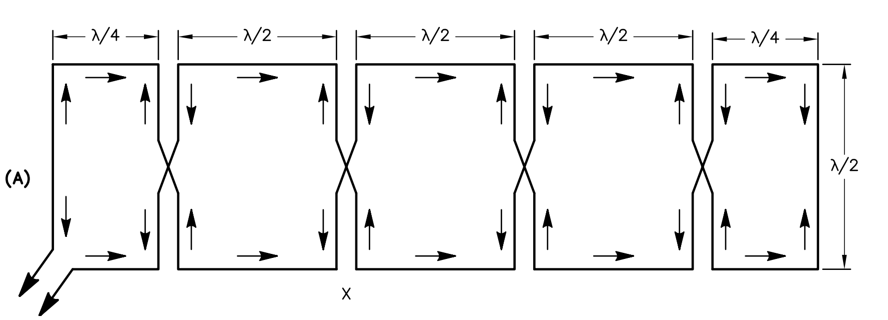


Fig 46—Typical Sterba array, an 8-element version.

ance over the band for a 40-meter double-extended Zepp is shown in **Fig 45**. This antenna is normally fed with open-wire transmission line to an antenna tuner. Other matching arrangements are, of course, possible. A method for transforming the feed-point impedance to 450 Ω and eliminating the minor lobes is given in Chapter 6.

THE STERBA ARRAY

Two collinear arrays can be combined to form the *Sterba array*, often called the *Sterba curtain*. An 8-element example of a Sterba array is shown in **Fig 46**. The four λ/4 elements joined on the ends are equivalent to two λ/2 elements. The two collinear arrays are spaced λ/2 and the λ/4 phasing lines connected together to provide λ/2 phasing lines. This arrangement has the advantage of increasing the gain for a given length and also increasing the E-plane directivity, which is no longer circular. An additional advantage of this array is that the wire forms a closed loop. For installations where icing is a problem a low voltage dc or low frequency (50 or 60 Hz) ac current can be passed through the wire to heat it for deicing. The heating current is isolated from RF by decoupling chokes. This is standard practice in commercial installations.

The number of sections in a Sterba array can be extended as far as desired but more than four or five are rarely used because of the slow increase in gain with extra elements, the narrow H-plane directivity and the appearance of multiple sidelobes. When fed at the point indicated the impedance is about 600 Ω. The antenna can also be fed at the point marked X. The impedance at this point will be about 1 kΩ. The gain of the 8-element array in Fig 46 will be between 7 to 8 dB over a single element.

Parallel Broadside Arrays

To obtain broadside directivity with parallel elements the currents in the elements must all be in phase. At a distant point lying on a line perpendicular to the axis of the array and also perpendicular to the plane containing the elements, the fields from all elements add up in phase. The situation is like that pictured in Fig 1 in this chapter, where four parallel $^1/_2$-λ dipoles were fed together a broadside array.

Broadside arrays of this type theoretically can have any number of elements. However, practical limitations of construction and available space usually limit the number of broadside parallel elements.

POWER GAIN

The power gain of a parallel-element broadside array depends on the spacing between elements as well as on the number of elements. The way in which the gain of a two-element array varies with spacing is shown in **Fig 47**. The greatest gain is obtained when the spacing is in the vicinity of 0.67 λ.

The theoretical gains of broadside arrays having more than two elements are approximately as follows:

No. of Parallel Elements	*dB Gain with λ/2 Spacing*	*dB Gain with 3/4 λ Spacing*
3	5.7	7.2
4	7.1	8.5
5	8.1	9.4
6	8.9	10.4

The elements must, of course, all lie in the same plane and all must be fed in phase.

DIRECTIVITY

The sharpness of the directive pattern depends on spacing between elements and number of elements. Larger element spacing will sharpen the main lobe, for a given number of elements, up to a point as was shown in Fig 39. The two-element array has no minor lobes when the spacing is λ/2, but small minor lobes appear at greater spacings. When three or more elements are used the pattern always has minor lobes.

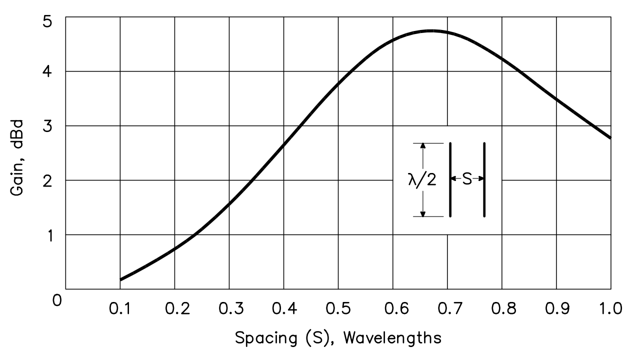


Fig 47—Gain as a function of the spacing between two parallel elements operated in phase (broadside).

Other Forms Of Broadside Arrays

For those who have the available room, multi-element arrays based on the broadside concept have something to offer. The antennas are large but of simple design and non-critical dimensions; they are also very economical in terms of gain per unit of cost.

Large arrays can often be fed at several different points. However, the pattern symmetry may be sensitive to the choice of feed-point within the array. Non-symmetrical feed points will result in small asymmetries in the pattern but these are not usually of great concern.

Arrays of three and four elements are shown in **Fig 48**. In the 3-element array with λ/2 spacing at A, the array is fed at the center. This is the most desirable point in that it tends to keep the power distribution among the elements uniform. However, the transmission line could alternatively be connected at either point B or C of Fig 48A, with only slight skewing of the radiation pattern.

When the spacing is greater than λ/2, the phasing lines must be 1 λ long and are not transposed between elements. This is shown Fig 48B. With this arrangement, any element spacing up to 1 λ can be used, if the phasing lines can be folded as suggested in the drawing.

The 4-element array at C is fed at the center of the system to make the power distribution among elements as uniform as possible. However, the transmission line could be connected at either point B, C, D or E. In this case the section of phasing line between B and D must be transposed to make the currents flow in the same direction in all elements. The 4-element array at C and the 3-element array at B have approximately the same gain when the element spacing in the array at B is $^3/_4$ λ.

An alternative feeding method is shown in Fig 48D. This system can also be applied to the 3-element arrays, and will result in better symmetry in any case. It is necessary only to move the phasing line to the center of each element, making connection to both sides of the line instead of one only.

The free-space pattern for a 4-element array with λ/2 spacing is shown in **Fig 49**. This is also approximately the pattern for a 3-element array with $^3/_4$ λ spacing.

Larger arrays can be designed and constructed by following the phasing principles shown in the drawings. No accurate figures are available for the impedances at the various feed points indicated in Fig 48. You can estimate it to be in the vicinity of 1 kΩ when the feed point is at a junction between the phasing line and a λ/2 element, becoming smaller as the number of elements in the array is increased. When the feed point is midway between end-fed elements as in Fig 48C, the feed-point impedance of a 4-element array is in the vicinity of 200 to 300 Ω, with 600 Ω open-wire phasing lines. The impedance at the feed point with the antenna shown at D should be about 1.5 kΩ.

NON-UNIFORM ELEMENT CURRENTS

The pattern for a 4-element broadside array shown in Fig 49 has substantial side lobes. This is typical for arrays more than λ/2 wide when equal currents flow in each element. Sidelobe amplitude can be reduced by using non-uniform current distribution among the elements. Many possible current amplitude distributions have been suggested. All of them have reduced current in the outer elements and greater current in the inner elements. This reduces the gain somewhat but can produce a more desirable pattern. One of the common current distributions is called *binomial current grading*. In this scheme the ratio of element currents is set equal to the coefficients of a polynomial. For example:

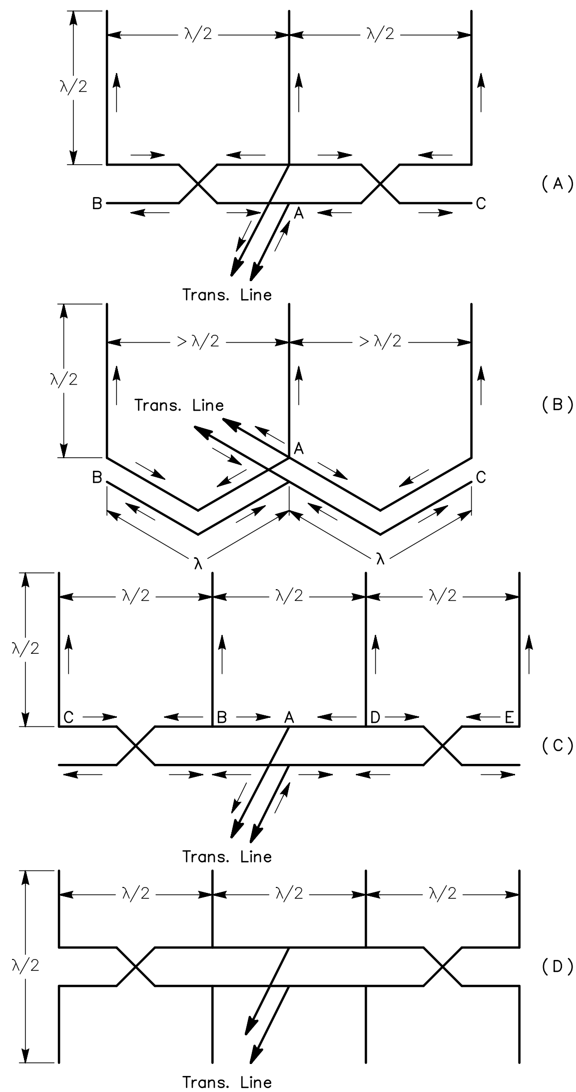


Fig 48—Methods of feeding three- and four-element broadside arrays with parallel elements.

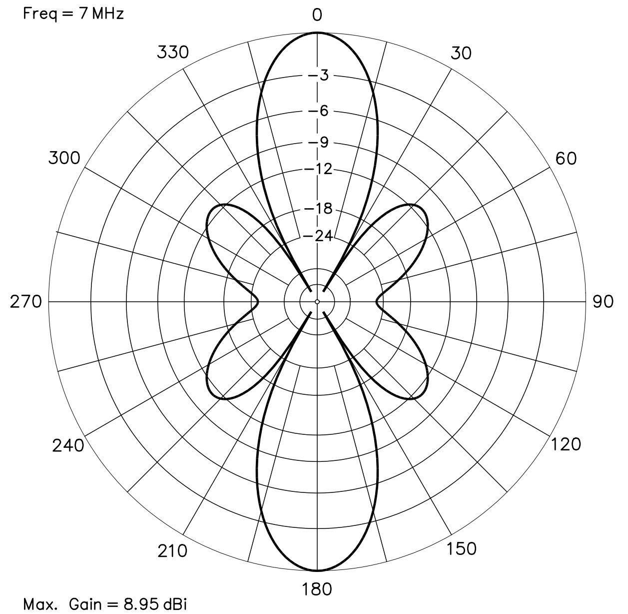


Fig 49—Free-space E-plane pattern of a four-element broadside array using parallel elements (Fig 48). This corresponds to the horizontal directive pattern at low wave angles for a vertically polarized array over ground. The axis of the elements lies along the 90°-270° line.

$$\begin{array}{llllll} x \quad 1 & 1x \quad 1 & 1{,}1 & & & \\ x \quad 1^{2} & 1x^{2} \quad 2x \quad 1, & 1{,}2{,}1 & & & \\ x \quad 1^{3} & 1x^{3} \quad 3x^{2} \quad 3x \quad 1, & 1{,}3{,}3{,}1 & & & \\ x \quad 1^{4} & 1x^{4} \quad 4x^{3} \quad 6x^{2} \quad 6x \quad 1, & 1{,}4{,}6{,}4{,}1 & & & \end{array} \qquad \text{(Eq 48)}$$

In a 2-element array the currents are equal, in a 3-element array the current in the center element is twice that in the outer elements, and so on.

HALF-SQUARE ANTENNA

On the low-frequency bands (40, 80 and 160 meters) it becomes increasingly difficult to use λ/2 elements because of their size. The half-square antenna is a 2-element broadside array with λ/4-high vertical elements and λ/2 horizontal spacing. See **Fig 50**. The free-space H-plane pattern for this array is shown in **Fig 51**. The antenna gives modest (4.2 dBi) but useful gain and has the advantage of only λ/4 height. Like all vertically polarized antennas, real-world performance depends directly on the characteristics of the ground surrounding it.

The half-square can be fed either at the point indicated or at the bottom end of one of the vertical elements using a voltage feed scheme, such as that shown in **Fig 52** for the bobtail curtain. The feed-point impedance is in the region of 50 Ω when fed at a corner as shown in Fig 50. A typical SWR plot is shown in **Fig 53**. Chapter 6 has a detailed discussion of the half-square antenna with several variations, together with practical considerations.

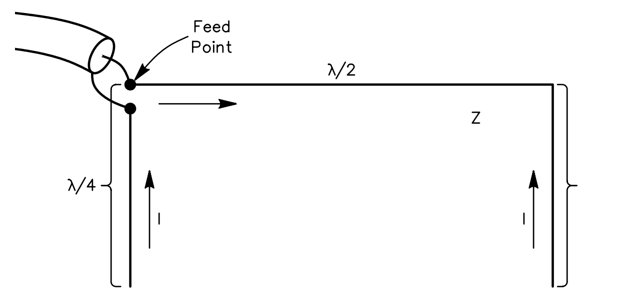


Fig 50—Layout for the half-square antenna.

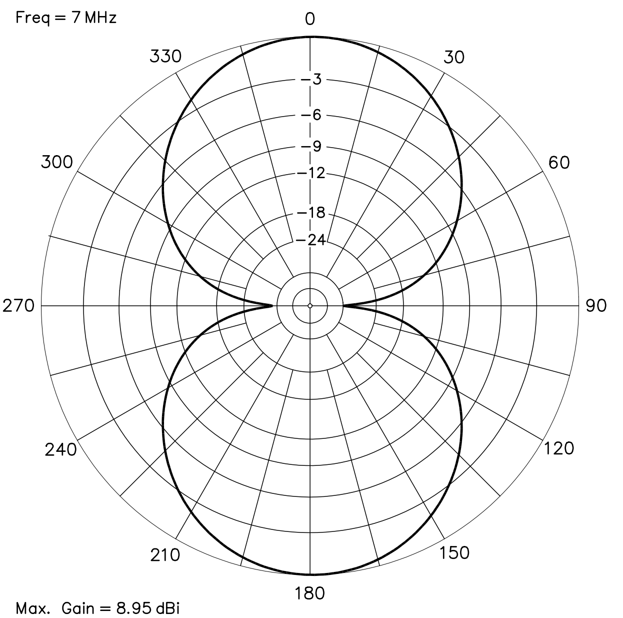


Fig 51—Free-space E-plane directive pattern for the half-square antenna.

BOBTAIL CURTAIN

The antenna system in Fig 52 uses the principles of co-phased verticals to produce a broadside, bidirectional pattern providing approximately 5.1 dB of gain over a single λ/4 element. The antenna performs as three in-phase, top-fed vertical radiators approximately λ/4 in height and spaced approximately λ/2. It is most effective for low-angle signals and makes an excellent long-distance antenna for 1.8, 3.5 or 7 MHz.

The three vertical sections are the actual radiating components, but only the center element is fed directly. The two horizontal parts, A, act as phasing lines and contribute very little to the radiation pattern. Because the current in the center element must be divided between the end sections, the current distribution approaches a binomial 1:2:1 ratio. The radiation pattern is shown in **Fig 54**.

The vertical elements should be as vertical as possible.

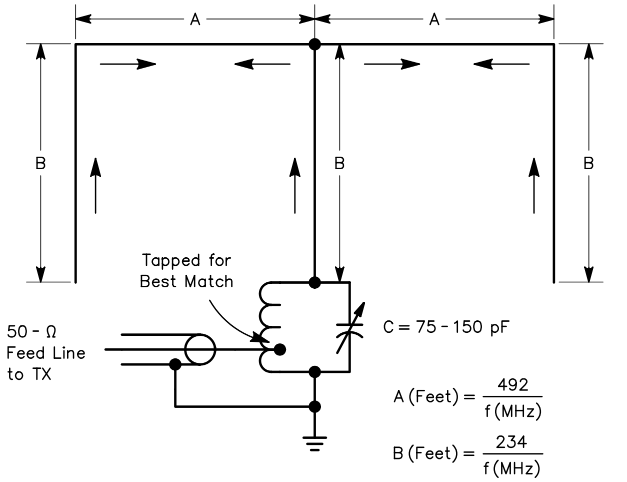


Fig 52—The bobtail curtain is an excellent low-angle radiator having broadside bidirectional characteristics. Current distribution is represented by the arrows. Dimensions A and B (in feet, for wire antennas) can be determined from the equations.

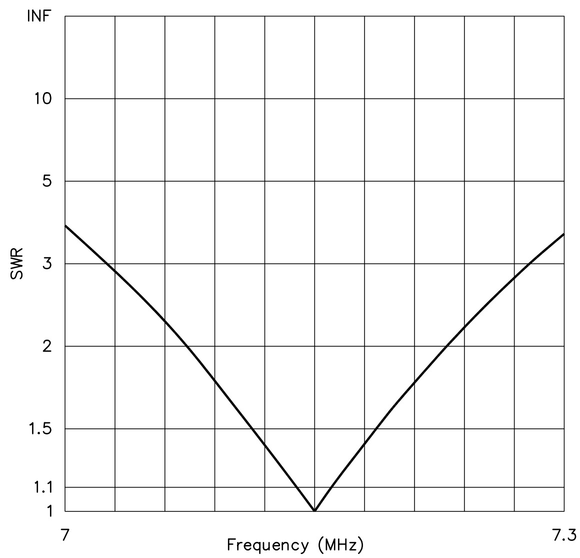


Fig 53—Typical SWR plot for a 40-meter half-square antenna fed at one corner. Antenna in free space.

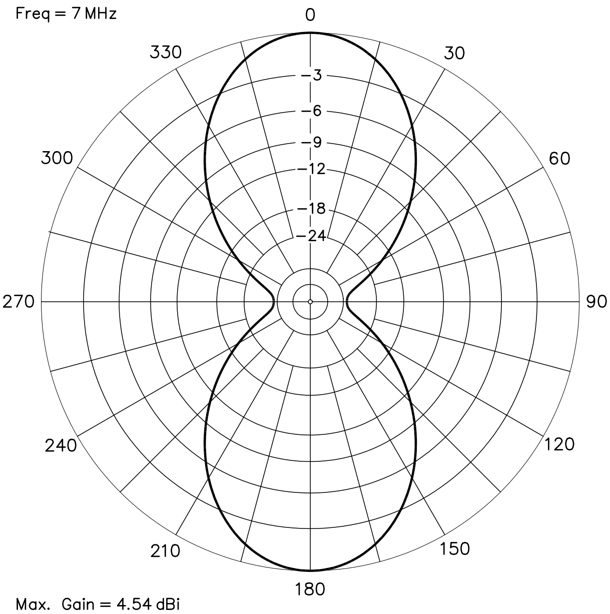


Fig 54—Calculated free-space E-plane directive diagram of the bobtail curtain shown in Fig 52. The array lies along the 90°-270° axis.

The height for the horizontal portion should be slightly greater than B, as shown in Fig 52. The tuning network is resonant at the operating frequency. The L/C ratio should be fairly low to provide good loading characteristics. As a starting point, a maximum capacitor value of 75 to 150 pF is recommended, and the inductor value is determined by C and the operating frequency. The network is first tuned to resonance and then the tap point is adjusted for the best match. A slight readjustment of C may be necessary. A link coil consisting of a few turns can also be used to feed the antenna.

A feeling for the matching bandwidth of this antenna can be obtained by looking at a feed point located at the top end of the center element. The impedance at this point will be approximately 32 Ω. An SWR plot (for $Z_0 = 32\ \Omega$) for an 80-meter bobtail curtain at this feed-point is shown in **Fig 55**. However, it is not advisable to actually connect a feedline at this point since it would detune the array and alter the pattern. This antenna is relatively narrow band. When fed at the bottom of the center element as shown in Fig 52, the SWR can be adjusted to be 1:1 at one frequency but the operating bandwidth for SWR < 2:1 may be even narrower than Fig 55 shows. For 80-meters, where operation is often desired in the CW DX window (3.510 MHz) and in the phone DX window (3.790 MHz), it will be necessary to retune the matching network as you change frequency. This can be done by switching a capacitor in or out, manually or remotely with a relay.

While the match bandwidth is quite narrow, the radiation pattern changes more slowly with frequency. **Fig 56** shows the variation in the pattern over the entire band (3.5 to 4.0 MHz). As would be expected, the gain increases with frequency because the antenna is larger in terms of wavelengths. The general shape of the pattern, however, is quite stable.

THE BRUCE ARRAY

Four variations of the Bruce array are shown in **Fig 57**. The Bruce is simply a wire folded so that the vertical sections carry large in-phase currents, while the horizontal sections carry small currents flowing in opposite directions with respect to the center of a section (indicated by dots). The radiation is vertically polarized. The gain is

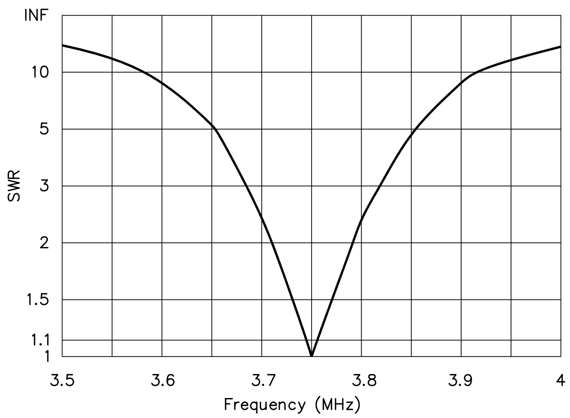


Fig 55—Typical SWR plot for an 80-meter bobtail curtain in free space. This is a narrow-band antenna.

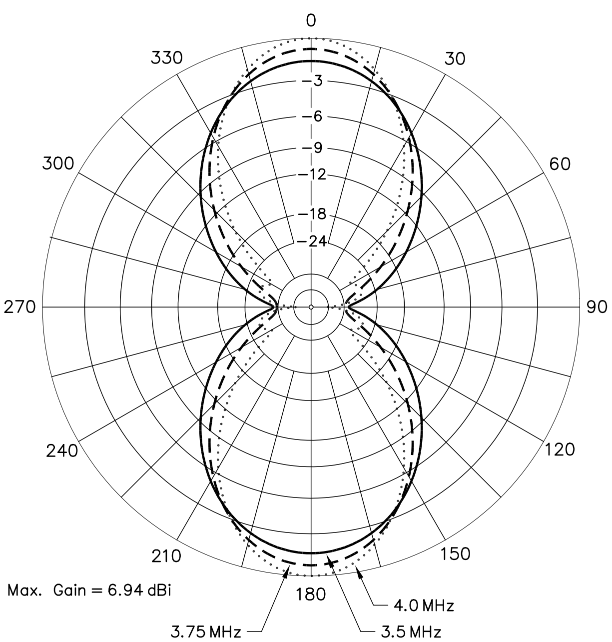


Fig 56—80-meter bobtail curtain's free-space E-plane pattern variation over the 80-meter band.

proportional to the length of the array but is somewhat smaller than you can obtain from a broadside array of λ/2 elements of the same length. This is because the radiating portion of the elements is only λ/4.

The Bruce array has a number of advantages:

1) The array is only λ/4 high. This is especially helpful on 80 and 160 meters, where the height of λ/2 supports becomes impractical for most amateurs.
2) The array is very simple. It is just a single piece of wire folded to form the array.
3) The dimensions of the array are very flexible. Depending on the available distance between supports, any number of elements can be used. The longer the array, the greater the gain.
4) The shape of the array does not have to be exactly 1.05 λ/4 squares. If the available height is short but the array can be made longer, then shorter vertical sections and longer horizontal sections can be used to maintain gain and resonance. Conversely, if more height is available but width is restricted then longer vertical sections can be used with shorter horizontal sections.
5) The array can be fed at other points more convenient for a particular installation.
6) The antenna is relatively low Q, so that the feed-point impedance changes slowly with frequency. This is very helpful on 80 meters, for example, where the antenna can be relatively broadband.
7) The radiation pattern and gain is stable over the width of an amateur band.

Note that the nominal dimensions of the array in Fig 57 call for section lengths = 1.05 λ/4. The need to use slightly longer elements to achieve resonance is common in large wire arrays. A quad loop behaves in the same manner. This is quite different from wire dipoles which are typically *shortened* by 2-5% to achieve resonance.

Fig 58 shows the variations in gain and pattern for 2- to 5-element 80-meter Bruce arrays. **Table 7** lists the gain over a vertical λ/2 dipole, a 4-radial ground-plane vertical and the size of the array. The gain and impedance parameters listed are for free space. Over real ground the patterns and gain will depend on the height above ground and the ground characteristics. Copper loss using #12 conductors is included.

Worthwhile gain can be obtained from these arrays,

Table 7
Bruce array length, impedance and gain as a function of number of elements

Number Elements	*Gain Over λ/2 Vertical Dipole*	*Gain over λ/4 Ground-Plane*	*Array Length Wavelengths*	*Approx. Feed Z, Ω*
2	1.2 dB	1.9 dB	1/4	130
3	2.8 dB	3.6 dB	1/2	200
4	4.3 dB	5.1 dB	3/4	250
5	5.3 dB	6.1 dB	1	300

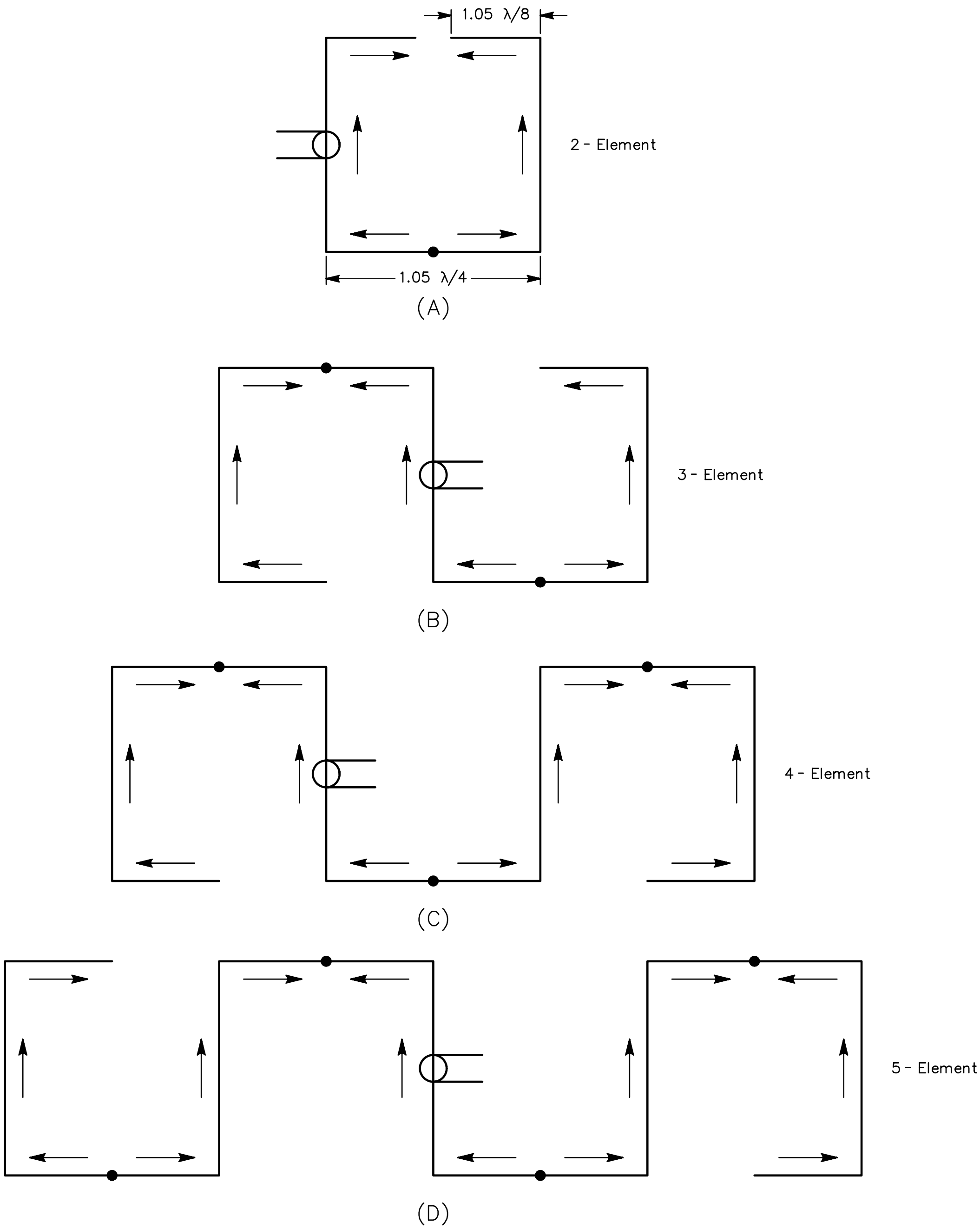


Fig 57—Various Bruce arrays: 2, 3, 4 and 5-element versions.

especially on 80 and 160 meters, where any gain is hard to come by. The feed-point impedance is for the center of a vertical section. From the patterns in Fig 58 you can see that sidelobes start to appear as the length of the array is increased beyond $^{3}/_{4}$ λ. This is typical for arrays using equal currents in the elements.

It is interesting to compare the bobtail curtain (Fig 52) with a 4-element Bruce array. **Fig 59** compares the radiation patterns for these two antennas. Even though the Bruce is shorter (3/4 λ) than the bobtail (1 λ), it has slightly more gain. The matching bandwidth is illustrated by the SWR curve in **Fig 60**. The 4-element Bruce has over twice the match bandwidth (200 kHz) than does the bobtail (75 kHz in Fig 55). Part of the gain difference is due to the binomial current distribution—the center element has twice the current as the outer elements in the bobtail. This reduces the gain slightly so that the 4-element Bruce becomes competitive. This is a good example of using more than the minimum number of elements to improve performance or to reduce size. On 160 meters the 4-element Bruce will be

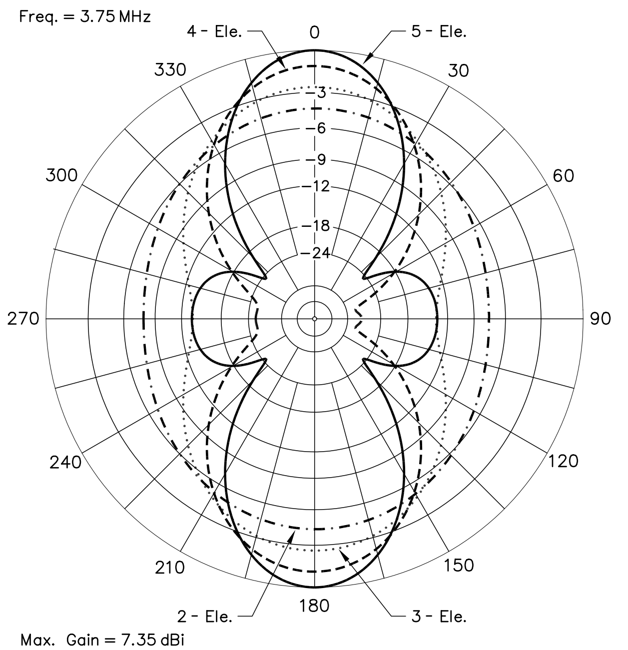


Fig 58—80-meter free-space E-plane directive patterns for the Bruce arrays shown in Fig 57. The 5-element's pattern is a solid line; the 4-element is a dashed line; the 3-element is a dotted line, and the 2-element version is a dashed-dotted line.

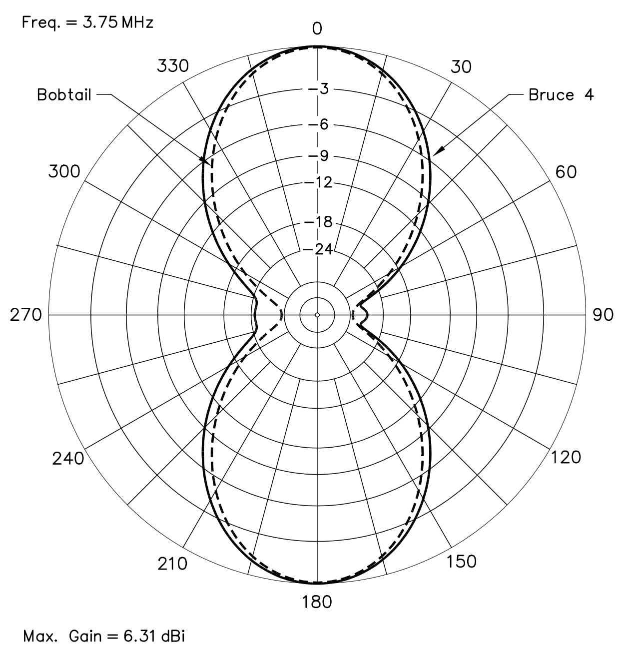


Fig 59—Comparison of free space patterns of a 4-element Bruce array (solid line) and a 3-element bobtail curtain (dashed line).

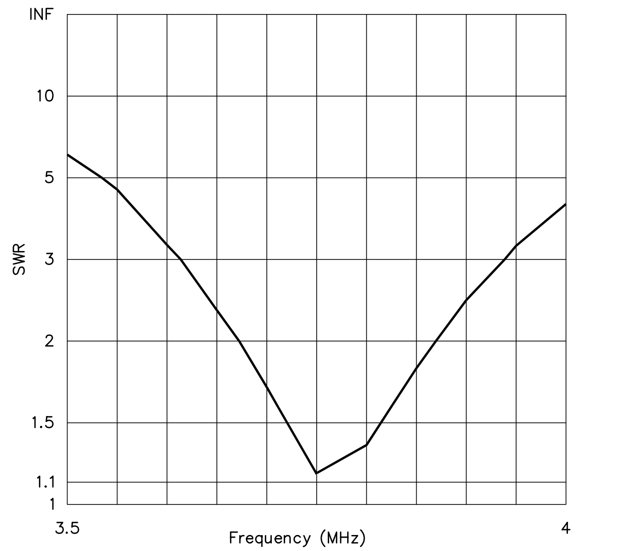


Fig 60—Typical SWR curve for a 4-element 80-meter Bruce array.

140 feet shorter than the bobtail, a significant reduction. If additional space is available for the bobtail (1λ) then a 5-element Bruce could be used, with a small increase in gain but also introducing some sidelobes.

The 2-element Bruce and the half-square antennas are both 2-element arrays. However, since the spacing between radiators is greater in the half-square (λ/2) the gain of the half-square is about 1 dB greater. If space is available, the half-square would be a better choice. If there is not room for a half-square then the Bruce, which is only half as long (λ/4), may be a good alternative. The 3-element Bruce, which has the same length (λ/2) as the half-square, has about 0.6 dB more gain than the half-square and will have a wider match bandwidth.

The Bruce antenna can be fed at many different points and in different ways. In addition to the feed points indicated in Fig 57, you may connect the feed line at the center of any of the vertical sections. In longer Bruce arrays, feeding at one end will result in some current imbalance among the elements but the resulting pattern distortion is small. Actually, the feed-point can be anywhere along a vertical section. One very convenient point is at an outside corner. The feed-point impedance will be higher (about 600 Ω). A good match for 450-Ω ladder-line can usually be found somewhere on the vertical section. It is important to recognize that feeding the antenna at a voltage node (dots in Fig 57) by breaking the wire and inserting an insulator, completely changes the current distribution. This will be discussed in the section on endfire arrays.

A Bruce can be fed unbalanced against ground or against a counterpoise as shown in **Fig 61**. Because it is a vertically polarized antenna, the better the ground system the better the performance. As few as two elevated radials can be used as shown in Fig 61B, but more radials can also

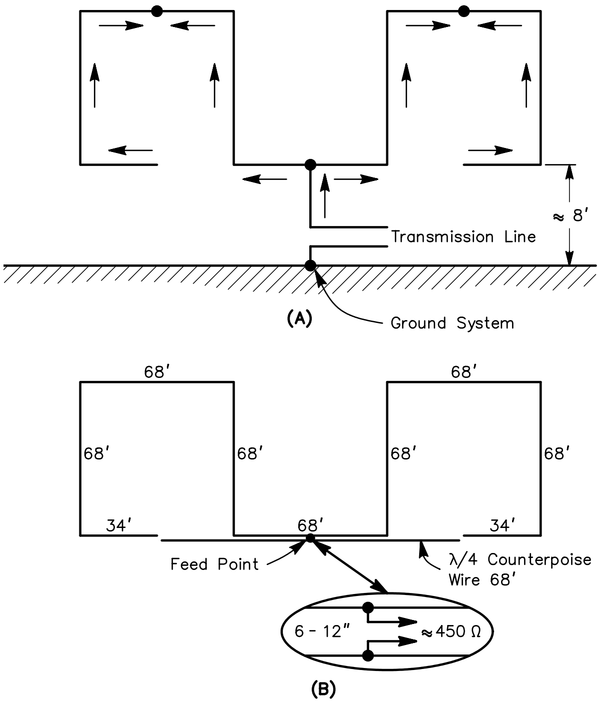


Fig 61—Alternate feed arrangements for the Bruce array. At A, the antenna is driven against a ground system and at B, it uses a two-wire counterpoise.

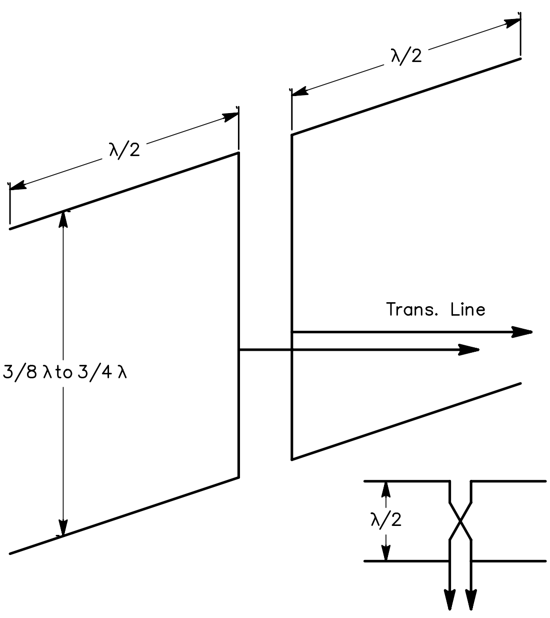


Fig 62—Four-element broadside array ("lazy H") using collinear and parallel elements.

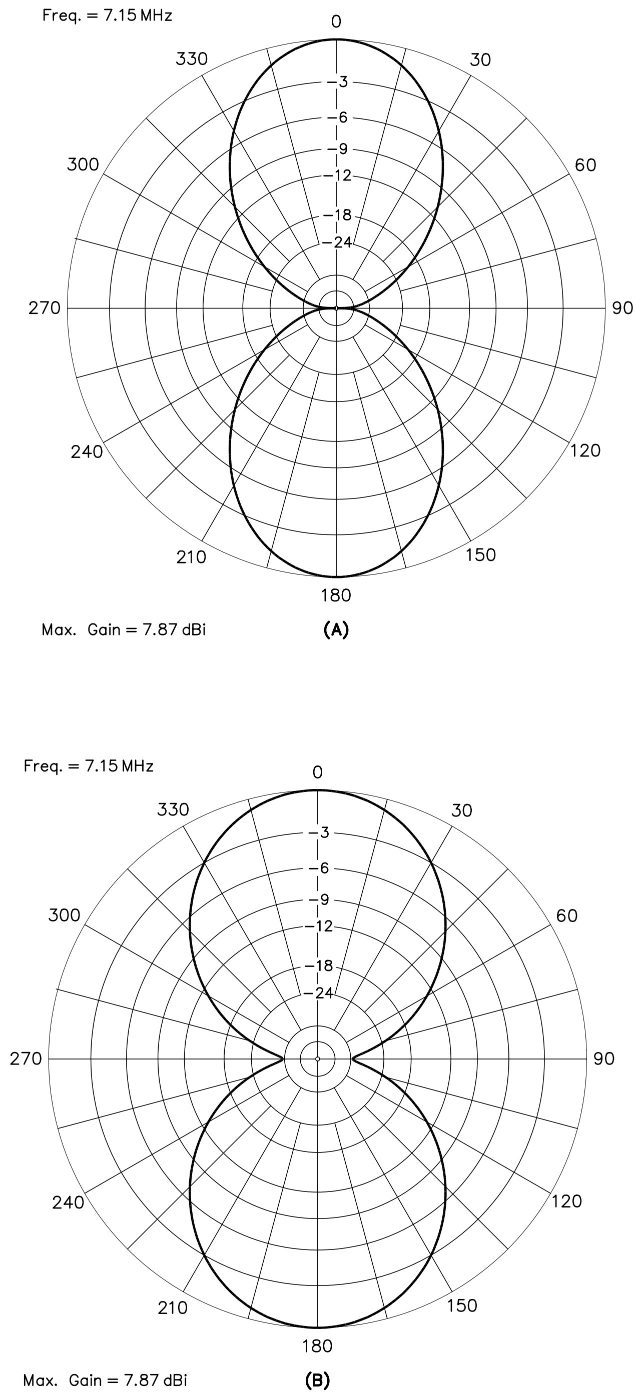


Fig 63—Free-space directive diagrams of the four-element antenna shown in Fig 62. At A is the E-plane pattern. The axis of the elements lies along the 90°-270° line. At B is the free-space H-plane pattern, viewed as if one set of elements is above the other from the ends of the elements.

be used to improve the performance, depending on local ground constants. The original development of the Bruce array in the late 1920s used this feed arrangement.

FOUR-ELEMENT BROADSIDE ARRAY

The 4-element array shown in **Fig 62** is commonly known as the *lazy H*. It consists of a set of two collinear elements and a set of two parallel elements, all operated in phase to give broadside directivity. The gain and directivity will depend on the spacing, as in the case of a simple parallel-element broadside array. The spacing may be chosen between the limits shown on the drawing, but spacings below $^3/_8$ λ are not worthwhile because the gain is small. Estimated gains compared to a single element are:

$^3/_8$ λ spacing—4.2 dB
$^1/_2$ λ spacing—5.8 dB
$^5/_8$ λ spacing—6.7 dB
$^3/_4$ λ spacing—6.3 dB

Half-wave spacing is generally used. Directive patterns for this spacing are given in **Figs 63** and **64**. With $^1/_2$ λ spacing between parallel elements, the impedance at the junction of the phasing line and transmission line is resistive and in the vicinity of 100 Ω. With larger or smaller spacing the impedance at this junction will be reactive as well as resistive. Matching stubs are recommended in cases where a non-resonant line is to be used. They may be calculated and adjusted as described in Chapter 26.

The system shown in Fig 62 may be used on two bands having a 2-to-1 frequency relationship. It should be designed for the higher of the two frequencies, using $^3/_4$ λ spacing between parallel elements. It will then operate on the lower frequency as a simple broadside array with $^3/_8$ λ spacing.

An alternative method of feeding is shown in the small diagram in Fig 62. In this case the elements and the phasing line must be adjusted exactly to an electrical half wavelength. The impedance at the feed point will be resistive and on the order of 2 kΩ.

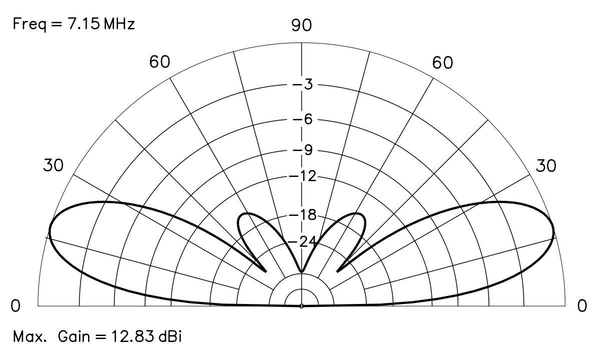


Fig 64—Vertical pattern of the four-element broadside antenna of Fig 62, when mounted with the elements horizontal and the lower set $^1/_2$ λ above flat ground. Stacked arrays of this type give best results when the lowest elements are at least $^1/_2$ λ high. The gain is reduced and the wave angle raised if the lowest elements are too close to ground.

THE BI-SQUARE ANTENNA

A development of the lazy H, known as the *bi-square antenna*, is shown in **Fig 65**. The gain of the bi-square is somewhat less than that of the lazy-H, but this array is attractive because it can be supported from a single pole. It has a circumference of 2 λ at the operating frequency, and is horizontally polarized.

The bi-square antenna consists of two 1λ radiators, fed 180° out of phase at the bottom of the array. The radiation resistance is 300 Ω, so it can be fed with either 300- or 600-Ω line. The free space gain of the antenna is about 5.8 dB, which is 3.7 dB more than a single dipole element. Gain may be increased by adding a parasitic reflector or director. Two bi-square arrays can be mounted at right angles and switched to provide omnidirectional coverage. In this way, the antenna wires may be used as part of the guying system for the pole.

Although it resembles a loop antenna, the bi-square is not a true loop because the ends opposite the feed point are open. However, identical construction techniques can be used for the two antenna types. Indeed, with a means of remotely closing the connection at the top for lower frequency operation, the antenna can be operated on two harmonically related bands. As an example, an array with 17 feet per side can be operated as a bi-square at 28 MHz and as a full-wave loop at 14 MHz. For two-band operation in this manner, the side length should favor the higher frequency. The length of a closed loop is not as critical.

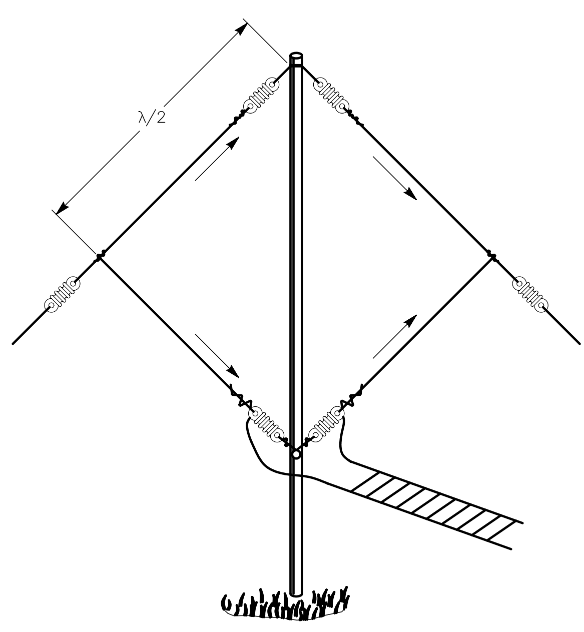


Fig 65—The bi-square array. It has the appearance of a loop, but is not a true loop because the conductor is open at the top. The length of each side, in feet, is 480/f (MHz).

End-Fire Arrays

The term *end-fire* covers a number of different methods of operation, all having in common the fact that the maximum radiation takes place along the array axis, and that the array consists of a number of parallel elements in one plane. End-fire arrays can be either bidirectional or unidirectional. In the bidirectional type commonly used by amateurs there are only two elements, and these are operated with currents 180° out of phase. Even though adjustment tends to be complicated, unidirectional end-fire driven arrays have also seen amateur use, primarily as a pair of phased, ground-mounted $^1/_4$ λ vertical elements. Extensive discussion of this array is contained in earlier sections of this chapter.

Horizontally polarized unidirectional end-fire arrays see little amateur use except in log-periodic arrays (described in Chapter 10). Instead, horizontally polarized unidirectional arrays usually have parasitic elements (described in Chapter 11).

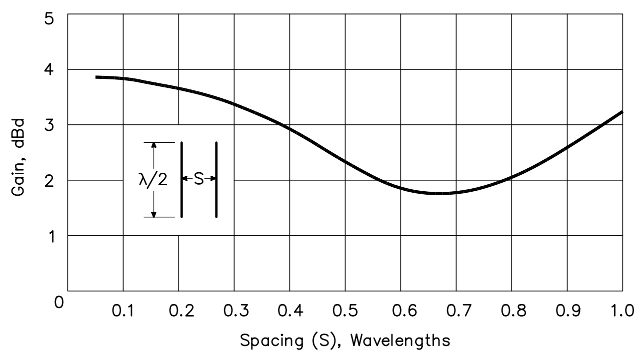


Fig 66—Gain of an end-fire array consisting of two elements fed 180° out of phase, as a function of the spacing between elements. Maximum radiation is in the plane of the elements and at right angles to them at spacings up to $^1/_2$ λ, but the direction changes at greater spacings.

TWO-ELEMENT END-FIRE ARRAY

In a 2-element array with equal currents out of phase, the gain varies with the spacing between elements as shown in **Fig 66**. The maximum gain occurs in the neighborhood of 0.1 λ spacing. Below that the gain drops rapidly due to conductor loss resistance.

The feed-point resistance for either element is very low at the spacings giving greatest gain, as shown in Fig 8 earlier in this chapter. The spacings most frequently used are $^1/_8$ and $^1/_4$ λ, at which the resistances of center-fed $^1/_2$ λ elements are about 9 and 32 Ω, respectively.

The effect of conductor resistance on gain for various spacings is shown in **Fig 67**. Because current along the element is not constant (it is approximately sinusoidal), the resistance shown is the equivalent resistance (Req) inserted at the center of the element to account for the loss distributed along the element.

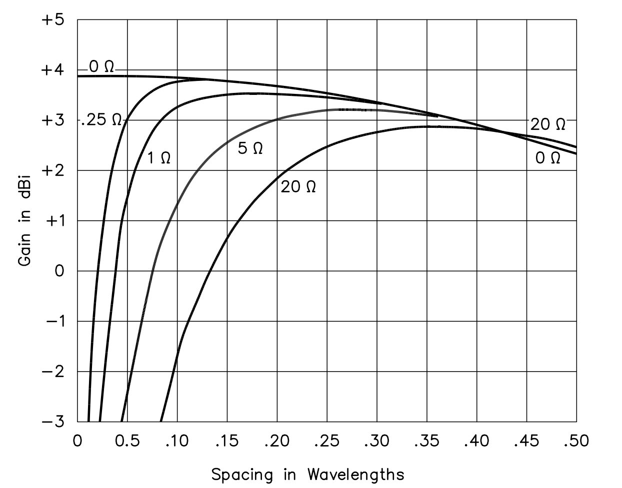


Fig 67—Gain over a single element of two out-of-phase elements in free space as a function of spacing for various loss resistances.

The equivalent resistance of a λ/2 element is $^1/_2$ the ac resistance (Rac) of the complete element. Rac is usually >> Rdc due to skin effect. For example, a 1.84 MHz dipole using #12 copper wire will have the following Req:

Wire length = 267 feet
Rdc = 0.00159 [Ω/foot] × 267 [feet] = 0.42 Ω
Fr = Rac/Rdc = 10.8
Req =(Rdc/2) × Fr = 2.29 Ω
For a 3.75 MHz dipole made with #12 wire, Req = 1.59 Ω.

In Fig 67, it is clear that end-fire antennas made with #12 or smaller wire will limit the attainable gain because of losses. There is no point in using spacings much less than 0.25 λ if you use wire elements. If instead you use elements made of aluminum tubing then smaller spacings can be used to increase gain. However, as the spacing is reduced below 0.25 λ the increase in gain is quite small even with good conductors. Closer spacings give little gain increase but can drastically reduce the operating bandwidth due to the rapidly increasing Q of the array.

Unidirectional End-Fire Arrays

Two parallel elements spaced $^1/_4$ λ apart and fed equal currents 90° out of phase will have a directional pattern in the plane at right angles to the plane of the array. See **Fig 68**. The maximum radiation is in the direction of the element in which the current lags. In the opposite direction the fields from the two elements cancel.

When the currents in the elements are neither in phase nor 180° out of phase, the feed-point resistances of the elements are not equal. This complicates the problem of feed-

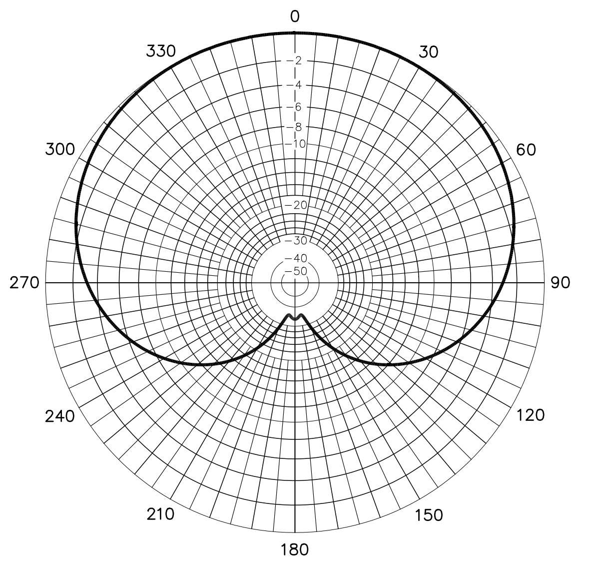


Fig 68—Representative H-plane pattern for a 2-element end-fire array with 90° spacing and phasing. The elements lie along the vertical axis, with the uppermost element the one of lagging phase. Dissimilar current distributions are taken into account. (Pattern computed with *ELNEC*.)

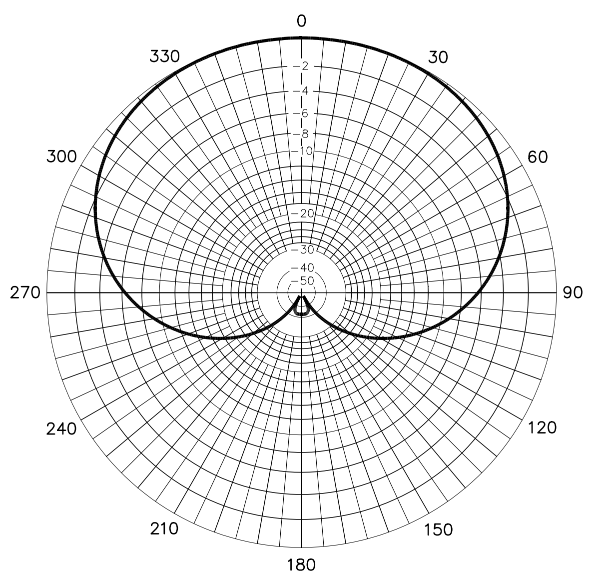


Fig 69—H-plane pattern for a 3-element end-fire array with binomial current distribution (the current in the center element is twice that in each end element). The elements are spaced $^1/_4$ λ apart along the 0°-180° axis. The center element lags the lower element by 90°, while the upper element lags the lower element by 180° in phase. Dissimilar current distributions are taken into account. (Pattern computed with *ELNEC*.)

ing equal currents to the elements, as discussed in earlier sections.

More than two elements can be used in a unidirectional end-fire array. The requirement for unidirectivity is that there must be a progressive phase shift in the element currents equal to the spacing, in electrical degrees, between the elements. The amplitudes of the currents in the various elements also must be properly related. This requires binomial current distribution. In the case of three elements, this requires that the current in the center element be twice that in the two outside elements, for 90° ($^1/_4$ λ) spacing and element current phasing. This antenna has an overall length of $^1/_2$ λ. The directive diagram is shown in **Fig 69**. The pattern is similar to that of Fig 68, but the 3-element binomial array has greater directivity, evidenced by the narrower half-power beamwidth (146° versus 176°). Its gain is 1.0 dB greater.

THE W8JK ARRAY

As pointed out earlier, John Kraus, W8JK, described his bi-directional flat-top W8JK beam antenna in 1940. See **Fig 70**. Two λ/2 elements are spaced λ/8 to λ/4 and driven 180° out of phase. The free-space radiation pattern for this antenna, using #12 copper wire, is given in **Fig 71**. The pattern is representative of spacings between λ/8 and λ/4 where the gain varies less than 0.5 dB. The gain over a dipole is about 3.3 dB (5.4 dBi referenced to an isotropic radiator), a worthwhile improvement. The feed-point impedance (including wire resistance) of *each* element is about 11 Ω for λ/8 spacing and 33 Ω for λ/4 spacing. The feed-point impedance at the center connection will depend on the length and Z_0 of the connecting transmission line.

Kraus gave a number of other variations for end-fire arrays, some of which are shown in **Fig 72**. The ones fed at the center (A, C and E) are usually horizontally polarized flat-top beams. The end-fed versions (B, D & F) are usually vertically polarized, where the feed point can be conveniently near ground. A practical variation of Fig 72B

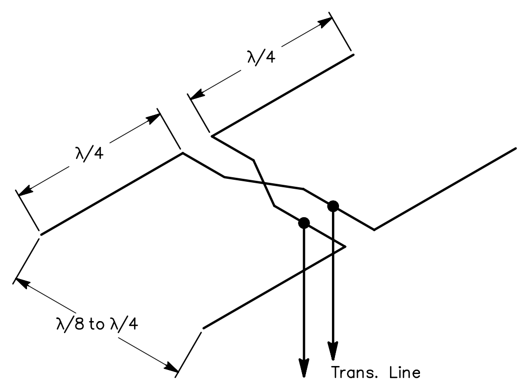


Fig 70—A 2-element W8JK array.

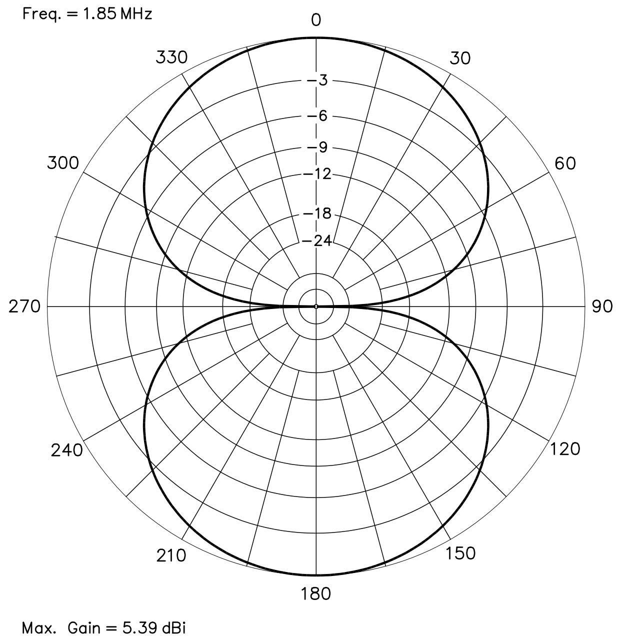


Fig 71—Free-space E-plane pattern for the 2-element W8JK array

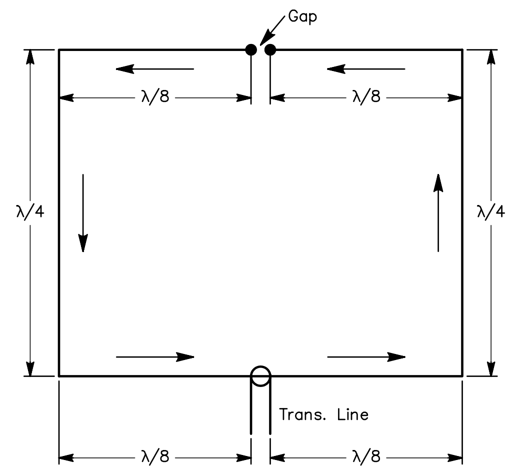


Fig 73—A 2-element end-fire array with reduced height.

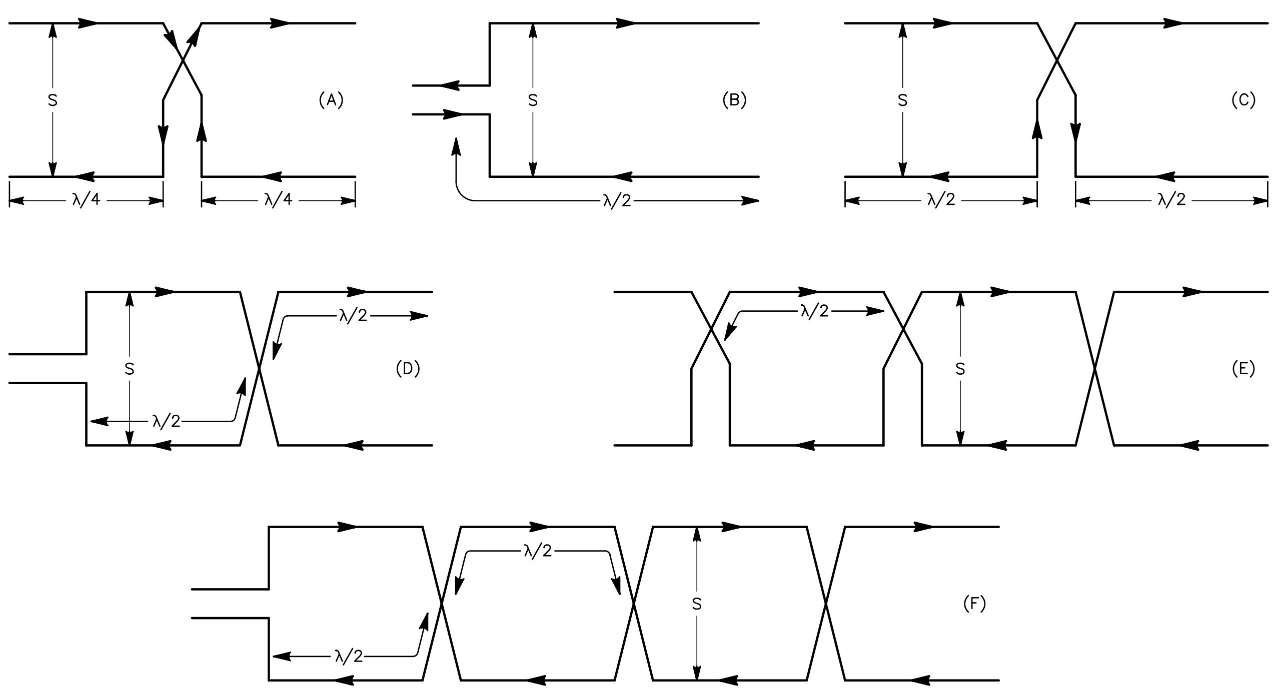


Fig 72—Six other variations of W8JK "flat-top beam" antennas.

is given in **Fig 73**. In this example, the height is limited to λ/4 so the ends can be bent over as shown, producing a 2-element end-fire array. This reduces the gain somewhat but allows much shorter supports, an important consideration on the low bands. If additional height is available, then you can achieve some additional gain. The upper ends can be bent over to fit the available height. The feed-point impedance will greater than 1 kΩ.

FOUR-ELEMENT END-FIRE AND COLLINEAR ARRAYS

The array shown in **Fig 74** combines collinear in-phase elements with parallel out-of-phase elements to give both broadside and end-fire directivity. It is a *two-section W8JK*. The approximate free-space gain using #12 copper wire is 4.9 dBi with $^{1}/_{8}$ λ spacing and 5.4 dBi with $^{1}/_{4}$ λ spacing. Directive patterns are given in **Figs 75** for free space, and in **Fig 76** for heights of 1 λ and $^{1}/_{2}$ λ above flat ground.

The impedance between elements at the point where the phasing line is connected is of the order of several thousand ohms. The SWR with an unmatched line consequently is quite high, and this system should be constructed with open-wire line (500 or 600 Ω) if the line is to be resonant. With λ/4 element spacing the SWR on a 600 Ω line is estimated to be in the vicinity of 3 or 4:1.

To use a matched line, you could connect a closed stub $^{3}/_{16}$ λ long at the transmission-line junction shown in Fig 74. The transmission line itself can then be tapped on this matching section at the point resulting in the lowest line SWR. This point can be determined by trial.

This type of antenna can be operated on two bands having a frequency ratio of 2 to 1, if a resonant feed line is used. For example, if you design for 28 MHz with λ/4 spacing between elements, you can also operate on 14 MHz as a simple 2-element end-fire array having λ/8 spacing.

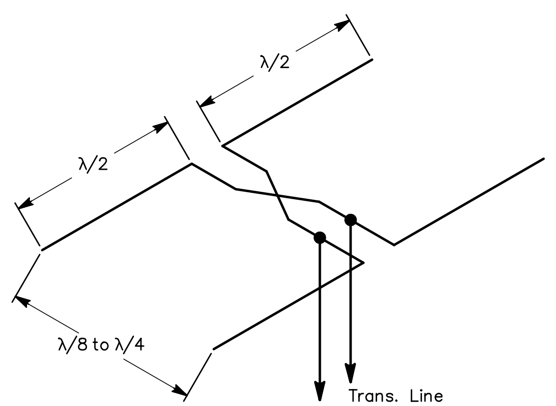


Fig 74—A four-element array combining collinear broadside elements and parallel end-fire elements, popularly known as a two-section W8JK array.

Combination Driven Arrays

You can readily combine broadside, end-fire and collinear elements to increase gain and directivity, and this is in fact usually done when more than two elements are used in an array. Combinations of this type give more gain, in a given amount of space, than plain arrays of the types just described. Since the combinations that can be worked out are almost endless, this section describes only a few of the simpler types.

The accurate calculation of the power gain of a multi-element array requires a knowledge of the mutual impedances

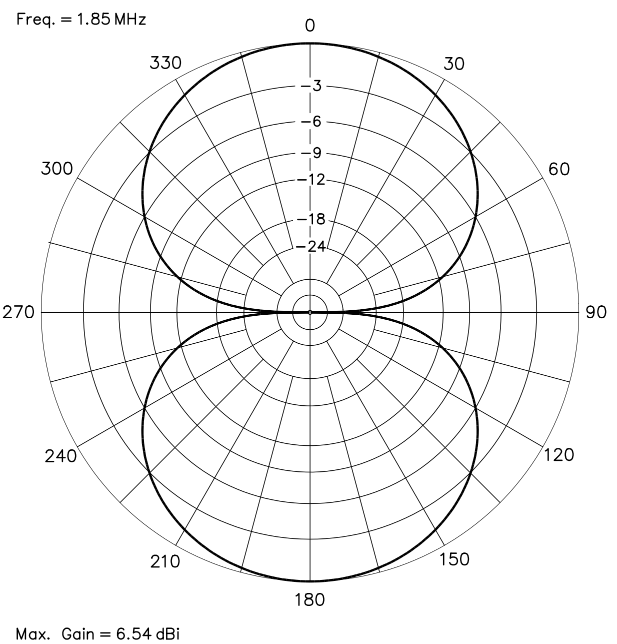


Fig 75—Free-space E-plane pattern for the antenna shown in Fig 74, with $^{1}/_{8}$-λ spacing. The elements are parallel to the 90°-270° line in this diagram. Less than a 1° change in half-power beamwidth results when the spacing is changed from $^{1}/_{8}$ to $^{1}/_{4}$ λ.

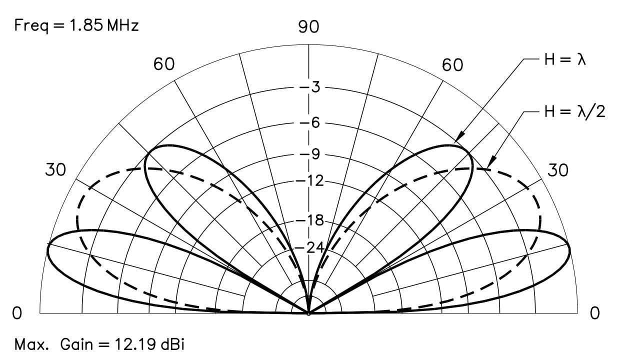


Fig 76—Elevation-plane pattern for the four-element antenna of Fig 74 when mounted horizontally at two heights over flat ground. Solid line = 1 λ high; dashed line =$^{1}/_{2}$ λ high.

between all elements, as discussed in earlier sections. For approximate purposes it is sufficient to assume that each set (collinear, broadside, end-fire) will have the gains as given earlier, and then simply add up the gains for the combination. This neglects the effects of cross-coupling between sets of elements. However, the array configurations are such that the mutual impedances from cross-coupling should be relatively small, particularly when the spacings are $\frac{1}{4}\lambda$ or more, so the estimated gain should be reasonably close to the actual gain. Alternatively, an antenna modeling program will give good estimates of all parameters for a real-world antenna, providing that you take care to model all applicable parameters.

FOUR-ELEMENT DRIVEN ARRAYS

The array shown in **Fig 77** combines parallel elements with broadside and end-fire directivity. The smallest array (physically)—$\frac{3}{8}\lambda$ spacing between broadside and $\frac{1}{8}\lambda$ spacing between end-fire elements—has an estimated gain of 6.5 dBi and the largest—$\frac{3}{4}$ and $\frac{1}{4}\lambda$ spacing, respectively—about 8.4 dBi. Typical directive patterns for a $\frac{1}{4} \times \frac{1}{2}\lambda$ array are given in **Figs 78** and **79**.

The impedance at the feed point will not be purely resistive unless the element lengths are correct and the phas-

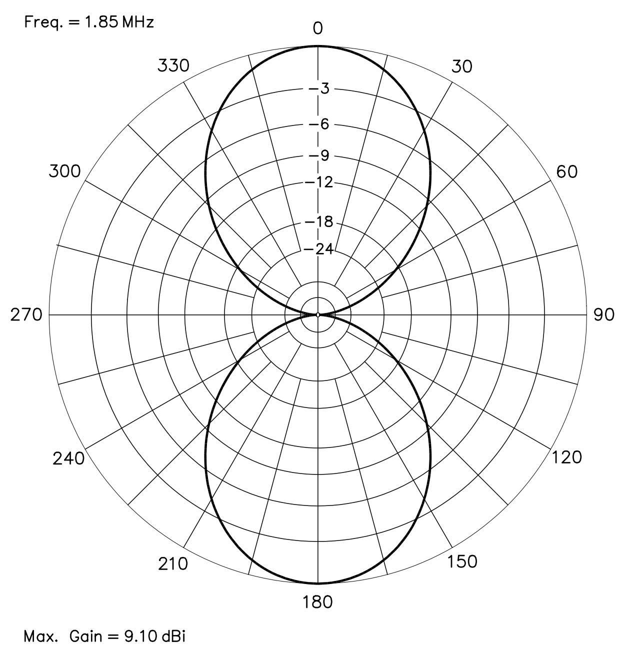


Fig 78—Free-space H-plane pattern of the four-element antenna shown in Fig 77.

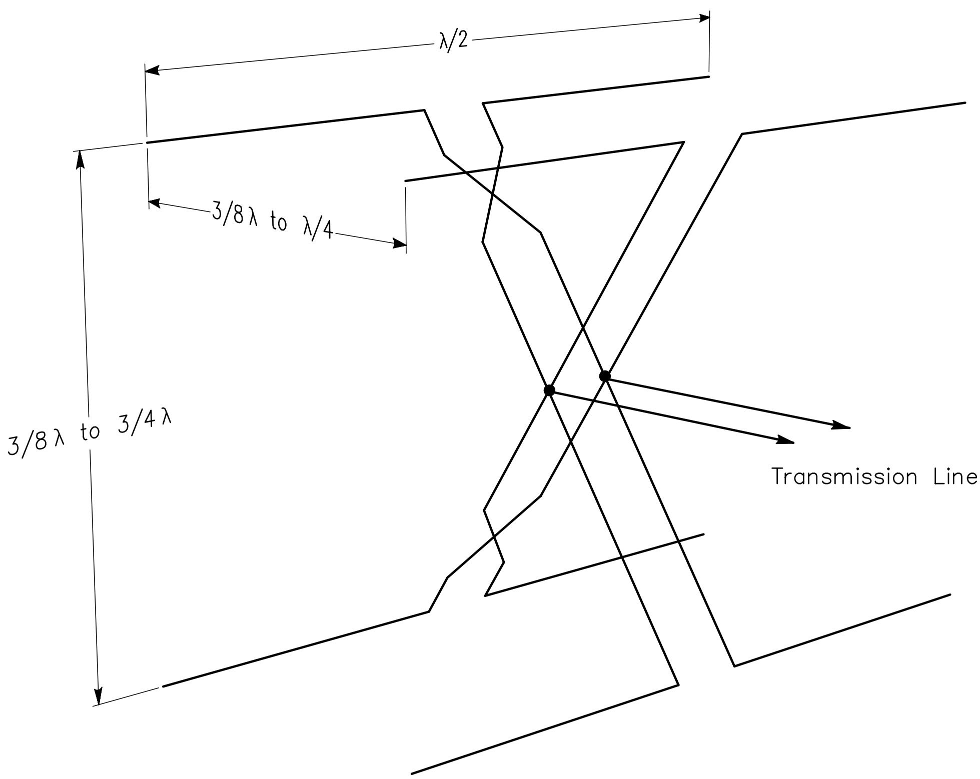


Fig 77—Four-element array combining both broadside and end-fire elements.

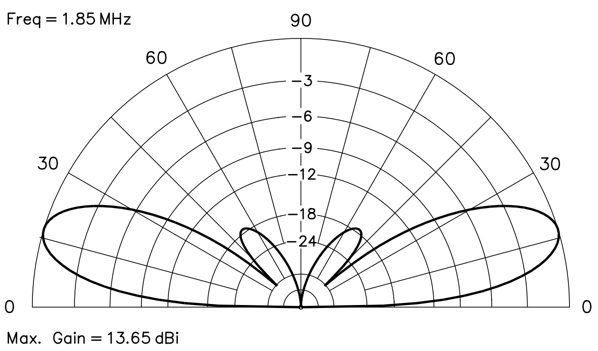


Fig 79—Vertical pattern of the antenna shown in Fig 77 at a mean height of $^3/_4$ λ (lowest elements $^1/_2$ λ above flat ground) when the antenna is horizontally polarized. For optimum gain and low wave angle the mean height should be at least $^3/_4$ λ.

ing lines are exactly $^1/_2$ λ long. (This requires somewhat less than $^1/_2$ λ spacing between broadside elements.) In this case the impedance at the junction is estimated to be over 10 kΩ. With other element spacings the impedance at the junction will be reactive as well as resistive, but in any event the SWR will be quite large. An open-wire line can be used as a resonant line, or a matching section may be used for non-resonant operation.

EIGHT-ELEMENT DRIVEN ARRAYS

The array shown in **Fig 80** is a combination of collinear and parallel elements in broadside and end-fire directivity. Common practice in a wire antenna is to use $^1/_2$ λ spacing for the parallel broadside elements and $^1/_4$ λ spacing for the end-fire elements. This gives a free-space gain of about 9.1 dBi. Directive patterns for an array using these spacings are similar to those of Figs 78 and 79, but are somewhat sharper.

The SWR with this arrangement will be high. Matching stubs are recommended for making the lines non-resonant. Their position and length can be determined as described in Chapter 26.

This system can be used on two bands related in frequency by a 2-to-1 ratio, providing it is designed for the higher of the two, with $^3/_4$ λ spacing between the parallel broadside elements and $^1/_4$ λ spacing between the end-fire elements. On the lower frequency it will then operate as a four-element antenna of the type shown in Fig 77, with $^3/_8$ λ broadside spacing and $^1/_8$ λ end-fire spacing. For two-band operation a resonant transmission line must be used.

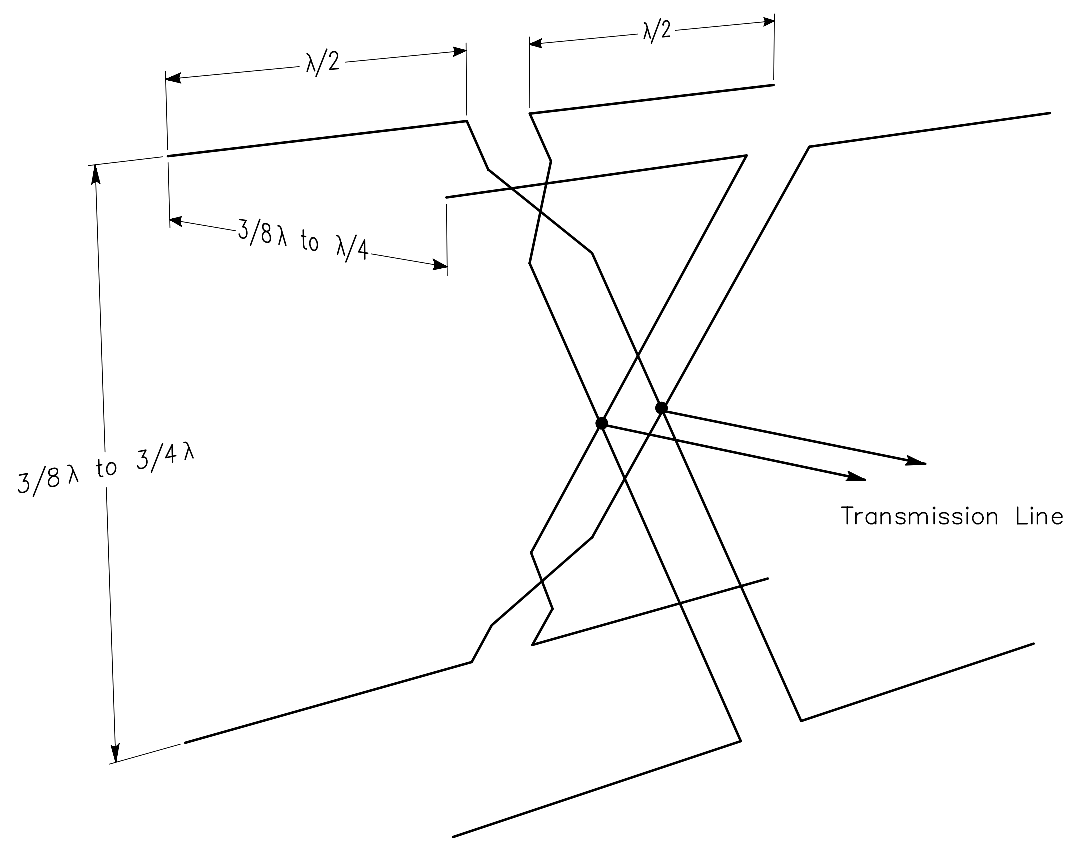


Fig 80—Eight-element driven array combining collinear and parallel elements for broadside and end-fire directivity.

PHASING ARROWS IN ARRAY ELEMENTS

In the antenna diagrams of preceding sections, the relative direction of current flow in the various antenna elements and connecting lines was shown by arrows. In laying out any antenna system it is necessary to know that the phasing lines are properly connected; otherwise the antenna may have entirely different characteristics than anticipated. The phasing may be checked either on the basis of current direction or polarity of voltages. There are two rules to remember:

1) In every $^1/_2\ \lambda$ section of wire, starting from an open end, the current directions reverse. In terms of voltage, the polarity reverses at each $^1/_2\ \lambda$ point, starting from an open end.
2) Currents in transmission lines always must flow in opposite directions in adjacent wires. In terms of voltage, polarities always must be opposite.

Examples of the use of current direction and voltage polarity are given at A and B, respectively, in **Fig 81**. The $\lambda/2$ points in the system are marked by small circles. When current in one section flows toward a circle, the current in the next section must also flow toward it, and vice versa. In the four-element antenna shown at A, the current in the upper right-hand element cannot flow toward the transmission line, because then the current in the right-hand section of the phasing line would have to flow upward and thus would be flowing in the same direction as the current in the left-hand wire. The phasing line would simply act like two wires in parallel in such a case. Of course, all arrows in the drawing could be reversed, and the net effect would be unchanged.

C shows the effect of transposing the phasing line. This transposition reverses the direction of current flow in the lower pair of elements, as compared with A, and thus changes the array from a combination collinear and end-fire arrangement into a collinear-broadside array.

The drawing at D shows what happens when the transmission line is connected at the center of a section of phasing line. Viewed from the main transmission line, the two parts of the phasing line are simply in parallel, so the half wavelength is measured from the antenna element along the upper section of phasing line and thence along the transmission line. The distance from the lower elements is measured in the same way. Obviously the two sections of phasing line should be the same length. If they are not, the current distribution becomes quite complicated; the element currents are neither in phase nor 180° out of phase, and the elements at opposite ends of the lines do not receive the same current. To change the element current phasing at D into the phasing at A, simply transpose the wires in one section of the phasing line; this reverses the direction of current flow in the antenna elements connected to that section of phasing line.

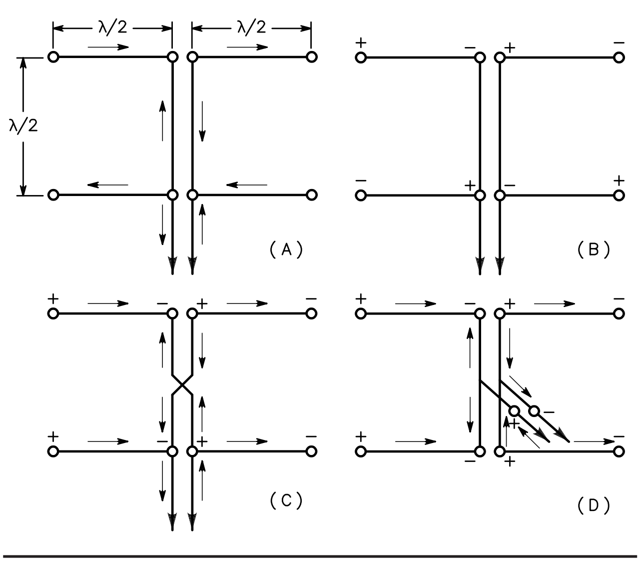


Fig 81—Methods of checking the phase of currents in elements and phasing lines.

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